Mathematics

Y12 to Y13 Mathematics Summer Independent Learning

June to August 2021

There are four tasks to work through.

Tasks 1 – 3 are compulsory (around 6 hours of work)

Task 4 is strongly recommended (around 4 – 6 hours of work)

Please read the following instructions very carefully and ensure you label and collate all your work ready for checking in September.

For your first Maths lesson please bring

- A large A4 folder with dividers.
- These instructions with the tables filled in (print out/copy the tables onto A4 paper).
- *Dated and titled work done on each of the topics listed in Task 1,2 and 3.*
- *A list of questions you need to ask prior to doing your initial test.*

Task 1: Consolidation of Pure Mathematics Y12 content (COMPULSORY)

1. Complete all questions, ideally in retrieval conditions.
2. Note down any topics struggled with, and use the below link to Jack Brown videos to help improve your understanding. https://sites.google.com/site/tlmaths314/home/a-level-maths-2017/full-a-level
3. Review any topics of particular concern using your gapped notes

Question / topic	Video(s) *(Tick)*	Gapped notes booklet used/ main issues?

Task 2 – Consolidation of further differentiation and further trigonometry (COMPULSORY)

1. Complete all questions from Trigonometry 'Sec, Cosec, Cot' worksheet
   https://www.mathsgenie.co.uk/resources/a-pure-sec-cosec-cot.pdf

2. Mark and correct all questions Trigonometry 'Sec, Cosec, Cot' worksheet
   https://www.mathsgenie.co.uk/resources/a-pure-sec-cosec-cotans.pdf

3. Complete all questions from Trigonometric identities worksheet
   https://www.mathsgenie.co.uk/resources/a-pure-trig-identities.pdf

4. Mark and correct all questions Trigonometric identities worksheet
   https://www.mathsgenie.co.uk/resources/a-pure-trig-identitiesans.pdf

5. Complete no less than 10 questions from Madasmaths further differentiation exam questions booklet (below).

   https://www.madasmaths.com/archive/maths_booklets/standard_topics/various/differentiation_ii_exam_questions.pdf

6. Mark and correct all questions using worked solutions within the same pdf. It may require significant zooming in to read worked solutions

7. Update review sheet with details of work completed.

Topic	**Score**	**Any points of particular weakness you need to address**
Introduction reciprocal Trigonometry (Sec, Cosec, Cot worksheet)	40	
Reciprocal Trigonometric identities	35	
Differentiation exam questions	N/A	
Total	75	

Task 3: Consolidation of Mechanics work (COMPULSORY)

1. Complete at least 2 questions from each section, this will be a minimum of 10 questions
2. Complete the exercise, showing well-structured algebraic methods. You do not need to complete every question in the exercise, but should make sure you get enough practice with both the skills themselves, and with setting out your reasoning clearly and logically.
3. Mark and correct your work.

Section	**Questions completed**		**Concepts / understanding you are having problems with**
Kinematics – Constant acceleration			
Kinematics – variable acceleration			
Forces – Non slopes			
Forces - slopes			

Task 4: (STRONGLY RECOMMENDED) past exam papers

Complete silver set A ; AS maths paper 1

https://crashmaths.com/a-level-practice-papers-edexcel/

Complete bronze set A ; AS maths paper 1

https://crashmaths.com/a-level-practice-papers-edexcel/

Task 1: Consolidation of Pure Mathematics Y12 content

Question 1

Show full workings

Write each of the following surd expressions as simple as possible.

a) $2\sqrt{32}+\sqrt{18}-3\sqrt{8}$.

b) $\dfrac{22}{4-\sqrt{5}}$.

Question 2

$$f(x)=3x^2+12x+8,\ x\in\mathbb{R}.$$

a) Express $f(x)$ in the form $a(x+b)^2+c$, where a, b and c are integers.

b) State the minimum value of $f(x)$.

c) Solve the equation $f(x)=0$, giving the answers as exact simplified surds.

Question 3

A cubic graph is defined by

$$f(x)\equiv x^3-3x^2-4x+12,\ x\in\mathbb{R}.$$

a) Show that $(x-3)$ is a factor of $f(x)$.

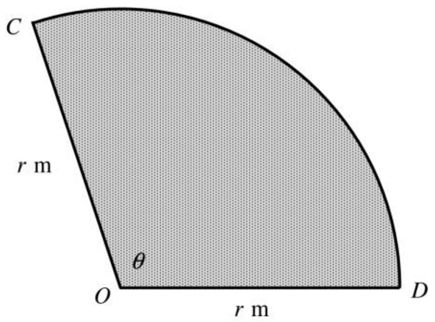
b) Hence factorize $f(x)$ as the product of three linear factors.

c) Sketch the graph of $f(x)$.

The sketch must include the coordinates of any points where the graph of $f(x)$ meets the coordinate axes.

d) State the roots of the equation $f(x-2)$

Question 4



A circular sector OCD, subtending an angle θ radians at its centre O, has a radius of r m.

The sector has an area of 0.25 m^2 and a perimeter of 2 m.

Determine the values of r and θ.

Question 5

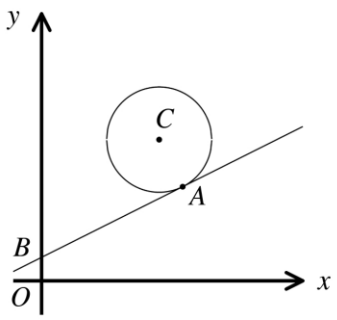
Solve each of the following equations, giving the final answers correct to three significant figures, where appropriate.

a) $6^{3x+2} = 30$.

b) $\log_4(12y+5) - \log_4(1-y) = 2$.

c) $8^{2t} - 8^t - 6 = 0$.

Question 6



The figure above shows a circle with centre at C with equation

$$x^2 + y^2 - 10x - 12y + 56 = 0.$$

The tangent to the circle at the point $A(6,4)$ meets the y axis at the point B.

a) Find an equation of the tangent to the circle at A.

b) Determine the area of the triangle ABC.

Question 7

Solve the following trigonometric equation in the range given.

$$4\tan^2\theta\cos\theta = 15,\ 0 \le \theta < 360°.$$

Question 8

Solve the following trigonometric equation in the range given.

$$1 + 2\sin(\theta + 25)° = 2.532, \quad 0 \le \theta < 360.$$

Question 9

$$f(x) = \sqrt{27x^3 + 1},\ x \ge -\tfrac{1}{3}.$$

The graph of $f(x)$ is stretched horizontally by scale factor 3, to produce the graph of $g(x)$.

Determine in its simplest form the equation of $g(x)$.

Question 10

(b) Without using your graphical calculator – Exam question could use (x +k) so important to know properly!

The curve C has equation

$$y = x^3 - 9x.$$

a) Sketch the graph of C.

b) Hence sketch on a separate diagram the graph of

$$y = (x+2)^3 - 9(x+2).$$

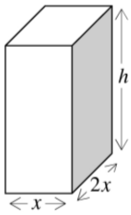
Each of the two sketches must include the coordinates of all the points where the curve meets the coordinate axes.

Question 11

$$y = 3 - \cos 2x°,\ 0 \le x \le 360.$$

Describe geometrically the three transformations that map the graph of $y = \cos x°$ onto the graph of $y = 3 - \cos 2x°$.

Question 12



The figure above shows the design of a fruit juice carton with capacity of 1000 cm^3.

The design of the carton is that of a closed cuboid whose base measures x cm by $2x$ cm, and its height is h cm.

a) Show that the surface area of the carton, $A \text{ cm}^2$, is given by

$$A = 4x^2 + \frac{3000}{x}.$$

b) Find the value of x for which A is stationary.

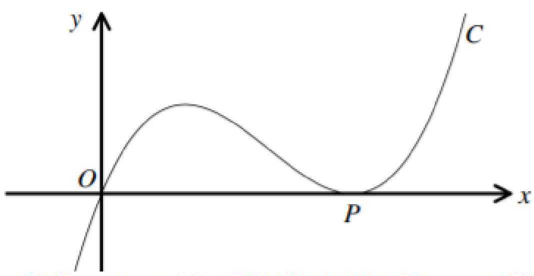
c) Calculate the minimum value for A, justifying fully the fact that it is indeed the minimum value of A.

Question 13

Use differentiation from first principles, to show that for $f(x) = x^4, x \in R$

$$f'(x) = 4x^3$$

Question 14



The figure above shows the cubic curve C which meets the coordinates axes at the origin O and at the point P.

The gradient function of C is given by

$$f'(x) = 3x^2 - 8x + 4.$$

a) Find an equation for C.

b) Determine the coordinates of P.

c) Calculate the area bounded by curve and the x-axis.

Question 15 – **Remember you can use column vectors or i,j,k notation!**

Relative to a fixed origin O, the points A, B and C have respective position vectors $2\mathbf{i}+3\mathbf{j}-\mathbf{k}$, $5\mathbf{i}-3\mathbf{j}+4\mathbf{k}$ and $7\mathbf{j}-4\mathbf{k}$.

a) Given that $ABCD$ is a parallelogram, determine the position vector of D.

b) Determine the distance AC and hence calculate the angle BAC

Question 16

The sum of the first 20 terms of an arithmetic series is 1070.

The sum of its fifth term and its tenth term is 65.

a) Find the first term and the common difference of the series.

b) Calculate the sum of the first 30 terms of the series.

Question 17

The second term of a geometric series is 4 and its sum to infinity is 18.

a) Show that the common ratio r of the series is a solution of the equation

$$9r^2-9r+2=0.$$

b) Find the two possible values of r and the corresponding values of the first term of the series.

The sum of the first n terms of the series is denoted by S_n.

c) Given that r takes the larger of the two values found in part (b) determine the smallest value of n for which S_n exceeds 17.975.

Question 18

$$y=\sqrt{4-12x}, \quad -\frac{1}{3}<x<\frac{1}{3}.$$

a) Find the binomial expansion of y in ascending powers of x up and including the term in x^3, writing all coefficients in their simplest form.

b) Hence find the coefficient of x^2 in the expansion of

$$(12x-4)(4-12x)^{\frac{1}{2}}.$$

Question 19

Given that

$$\frac{2x^3 + x^2 - 4x + 1}{x^2 + x - 2} \equiv Ax + B + \frac{C}{x + D},$$

use polynomial division, or another appropriate method, to find the value of each of the constants A, B, C and D.

Task 2 – **Consolidation of further differentiation and further trigonometry**

Links to questions/answers

1. Trigonometry 'Sec, Cosec, Cot' worksheet
   https://www.mathsgenie.co.uk/resources/a-pure-sec-cosec-cot.pdf

2. Solutions to Trigonometry 'Sec, Cosec, Cot' worksheet
   https://www.mathsgenie.co.uk/resources/a-pure-sec-cosec-cotans.pdf

3. Trigonometric identities worksheet
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4. Solutions to Trigonometric identities worksheet
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5. Madasmaths further differentiation exam questions booklet

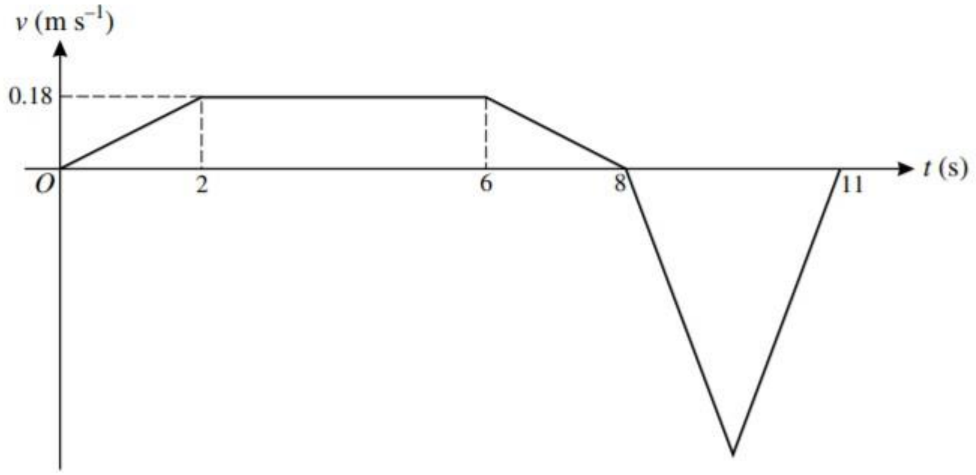
   https://www.madasmaths.com/archive/maths_booklets/standard_topics/various/differentiation_ii_exam_questions.pdf

[Task 3 questions on next page]

Task 3: Consolidation of Mechanics work

Kinematics – Constant acceleration

Standard



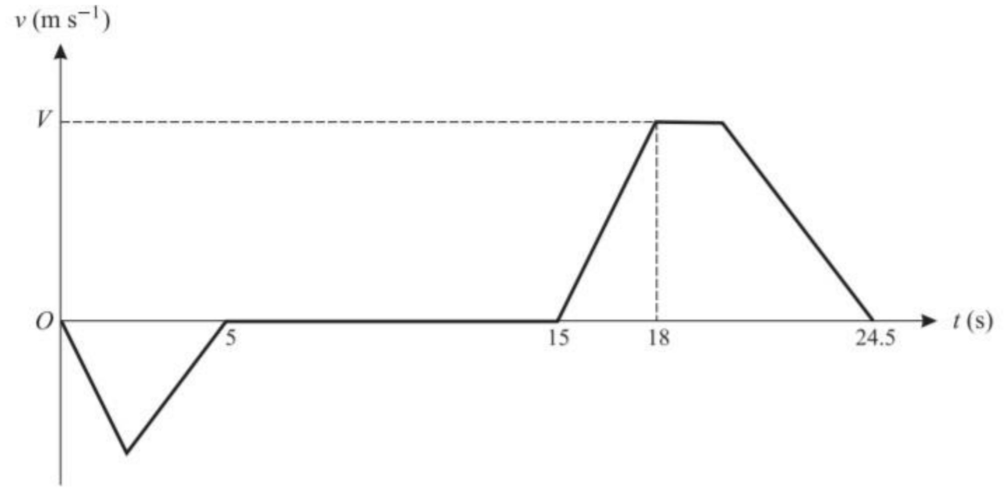
The diagram shows the velocity-time graph for the motion of a machine's cutting tool. The graph consists of five straight line segments. The tool moves forward for 8 s while cutting and then takes 3 s to return to its starting position. Find

(i) the acceleration of the tool during the first 2 s of the motion, [1]

(ii) the distance the tool moves forward while cutting, [2]

(iii) the greatest speed of the tool during the return to its starting position. [2]

Challenging



The diagram shows the velocity-time graph for a lift moving between floors in a building. The graph consists of straight line segments. In the first stage the lift travels downwards from the ground floor for 5 s, coming to rest at the basement after travelling 10 m.

(i) Find the greatest speed reached during this stage. [2]

The second stage consists of a 10 s wait at the basement. In the third stage, the lift travels upwards until it comes to rest at a floor 34.5 m above the basement, arriving 24.5 s after the start of the first stage. The lift accelerates at 2 m s^{-2} for the first 3 s of the third stage, reaching a speed of V m s^{-1}. Find

(ii) the value of V, [2]

(iii) the time during the third stage for which the lift is moving at constant speed, [3]

(iv) the deceleration of the lift in the final part of the third stage. [2]

Hard

The top of a cliff is 40 metres above the level of the sea. A man in a boat, close to the bottom of the cliff, is in difficulty and fires a distress signal vertically upwards from sea level. Find

(i) the speed of projection of the signal given that it reaches a height of 5 m above the top of the cliff, [2]

(ii) the length of time for which the signal is above the level of the top of the cliff. [2]

The man fires another distress signal vertically upwards from sea level. This signal is above the level of the top of the cliff for $\sqrt{(17)}$ s.

(iii) Find the speed of projection of the second signal. [3]

Kinematics - variable acceleration

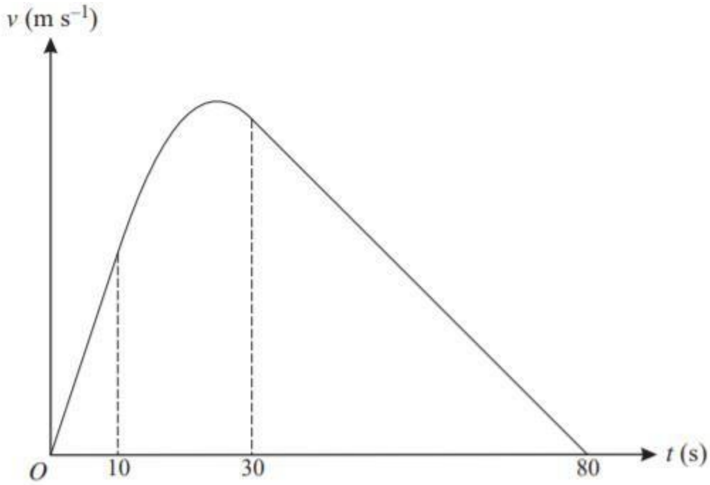
Standard

A particle P moves along the x-axis in the positive direction. The velocity of P at time t s is $0.03t^2$ m s^{-1}. When $t = 5$ the displacement of P from the origin O is 2.5 m.

(i) Find an expression, in terms of t, for the displacement of P from O. [4]

(ii) Find the velocity of P when its displacement from O is 11.25 m. [3]

Challenging



An object P travels from A to B in a time of 80 s. The diagram shows the graph of v against t, where v m s^{-1} is the velocity of P at time t s after leaving A. The graph consists of straight line segments for the intervals $0 \leqslant t \leqslant 10$ and $30 \leqslant t \leqslant 80$, and a curved section whose equation is $v = -0.01t^2 + 0.5t - 1$ for $10 \leqslant t \leqslant 30$. Find

(i) the maximum velocity of P, [4]

(ii) the distance AB. [9]

Hard

A vehicle is moving in a straight line. The velocity v m s^{-1} at time t s after the vehicle starts is given by

$$v = A(t - 0.05t^2) \quad \text{for } 0 \leqslant t \leqslant 15,$$

$$v = \frac{B}{t^2} \quad \text{for } t \geqslant 15,$$

where A and B are constants. The distance travelled by the vehicle between $t = 0$ and $t = 15$ is 225 m.

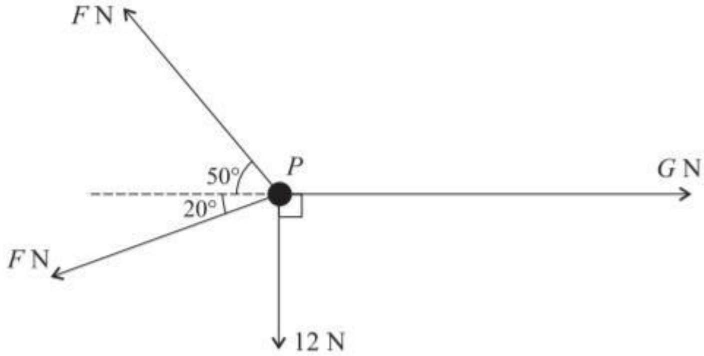
(i) Find the value of A and show that $B = 3375$. [5]

(ii) Find an expression in terms of t for the total distance travelled by the vehicle when $t \geqslant 15$. [3]

(iii) Find the speed of the vehicle when it has travelled a total distance of 315 m. [3]

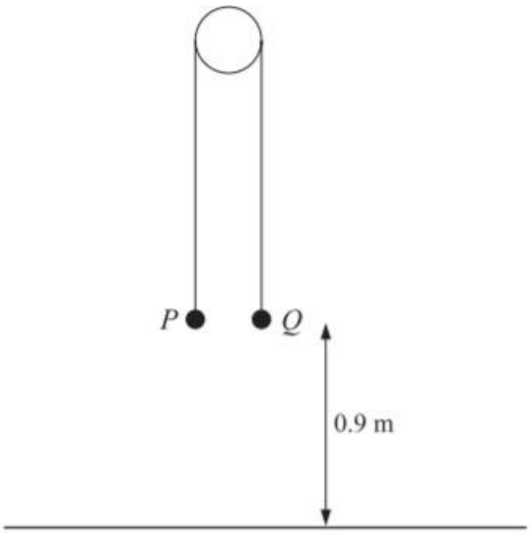
Forces - Non – slopes

Question 1



A particle P is in equilibrium on a smooth horizontal table under the action of horizontal forces of magnitudes F N, F N, G N and 12 N acting in the directions shown. Find the values of F and G. [6]

Question 2



Particles P and Q, of masses 0.6 kg and 0.2 kg respectively, are attached to the ends of a light inextensible string which passes over a smooth fixed peg. The particles are held at rest with the string taut. Both particles are at a height of 0.9 m above the ground (see diagram). The system is released and each of the particles moves vertically. Find

(i) the acceleration of P and the tension in the string before P reaches the ground, [5]

(ii) the time taken for P to reach the ground. [2]

Question 3

A car is towing a trailer along a straight horizontal road by means of a horizontal tow-rope. The mass of the car is 1400 kg. The mass of the trailer is 700 kg. The car and the trailer are modelled as particles and the tow-rope as a light inextensible string. The resistances to motion of the car and the trailer are assumed to be constant and of magnitude 630 N and 280 N respectively. The driving force on the car, due to its engine, is 2380 N. Find

(a) the acceleration of the car,

(3)

(b) the tension in the tow-rope.

(3)

When the car and trailer are moving at 12 m s^{-1}, the tow-rope breaks. Assuming that the driving force on the car and the resistances to motion are unchanged,

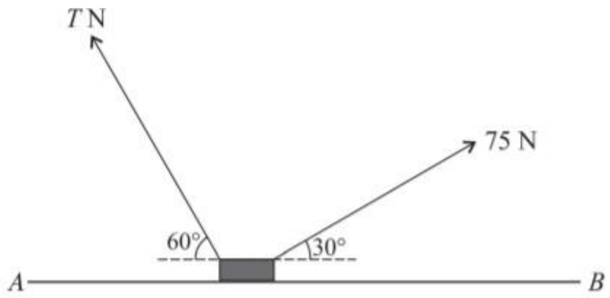
(c) find the distance moved by the car in the first 4 s after the tow-rope breaks.

(6)

(d) State how you have used the modelling assumption that the tow-rope is inextensible.

(1)

Question 4

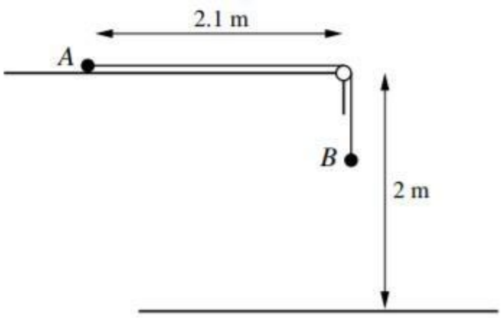


Two light strings are attached to a block of mass 20 kg. The block is in equilibrium on a horizontal surface AB with the strings taut. The strings make angles of 60° and 30° with the horizontal, on either side of the block, and the tensions in the strings are T N and 75 N respectively (see diagram).

(i) Given that the surface is smooth, find the value of T and the magnitude of the contact force acting on the block. [5]

(ii) It is given instead that the surface is rough and that the block is on the point of slipping. The frictional force on the block has magnitude 25 N and acts towards A. Find the coefficient of friction between the block and the surface. [6]

Question 5



Particles A and B, of masses 0.2 kg and 0.45 kg respectively, are connected by a light inextensible string of length 2.8 m. The string passes over a small smooth pulley at the edge of a rough horizontal surface, which is 2 m above the floor. Particle A is held in contact with the surface at a distance of 2.1 m from the pulley and particle B hangs freely (see diagram). The coefficient of friction between A and the surface is 0.3. Particle A is released and the system begins to move.

(i) Find the acceleration of the particles and show that the speed of B immediately before it hits the floor is 3.95 m s^{-1}, correct to 3 significant figures. [7]

(ii) Given that B remains on the floor, find the speed with which A reaches the pulley. [4]

Forces - Slopes

Question 1

A block, of mass 5 kg, slides down a rough plane inclined at 40° to the horizontal. When modelling the motion of the block, assume that there is no air resistance acting on it.

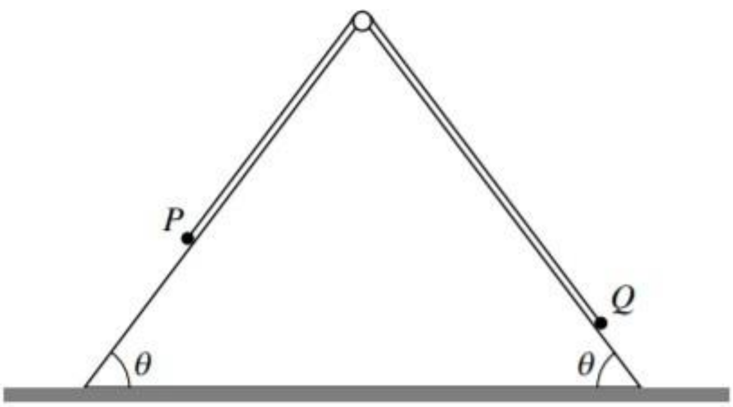
(a) Draw and label a diagram to show the forces acting on the block. *(1 mark)*

(b) Show that the magnitude of the normal reaction force acting on the block is 37.5 N, correct to three significant figures. *(2 marks)*

(c) Given that the acceleration of the block is $0.8\,\mathrm{m\,s^{-2}}$, find the coefficient of friction between the block and the plane. *(6 marks)*

(d) In reality, air resistance does act on the block. State how this would change your value for the coefficient of friction and explain why. *(2 marks)*

Question 2



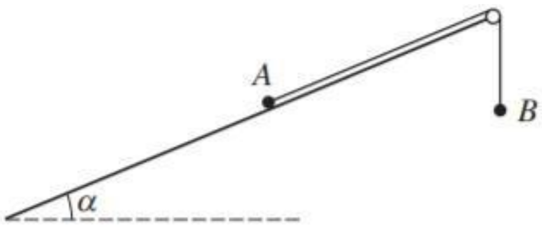
Particles P and Q, of masses 0.6 kg and 0.4 kg respectively, are attached to the ends of a light inextensible string. The string passes over a small smooth pulley which is fixed at the top of a vertical cross-section of a triangular prism. The base of the prism is fixed on horizontal ground and each of the sloping sides is smooth. Each sloping side makes an angle θ with the ground, where $\sin\theta = 0.8$. Initially the particles are held at rest on the sloping sides, with the string taut (see diagram). The particles are released and move along lines of greatest slope.

(i) Find the tension in the string and the acceleration of the particles while both are moving. [5]

The speed of P when it reaches the ground is $2\,\mathrm{m\,s^{-1}}$. On reaching the ground P comes to rest and remains at rest. Q continues to move up the slope but does not reach the pulley.

(ii) Find the time taken from the instant that the particles are released until Q reaches its greatest height above the ground. [4]

Question 3



A light inextensible string has a particle A of mass 0.26 kg attached to one end and a particle B of mass 0.54 kg attached to the other end. The particle A is held at rest on a rough plane inclined at angle α to the horizontal, where $\sin\alpha = \frac{5}{13}$. The string is taut and parallel to a line of greatest slope of the plane. The string passes over a small smooth pulley at the top of the plane. Particle B hangs at rest vertically below the pulley (see diagram). The coefficient of friction between A and the plane is 0.2. Particle A is released and the particles start to move.

(i) Find the magnitude of the acceleration of the particles and the tension in the string. [6]

Particle A reaches the pulley 0.4 s after starting to move.

(ii) Find the distance moved by each of the particles. [2]

[Solutions begin on next page]

Task 1 solutions

Question 1

(a) $2\sqrt{32}+\sqrt{18}-3\sqrt{8}$

$= 2\sqrt{16\times 2}+\sqrt{9\times 2}-3\sqrt{4\times 2}$

$= 2\times 4\sqrt{2}+3\sqrt{2}-3\times 2\sqrt{2}$

$= 8\sqrt{2}+3\sqrt{2}-6\sqrt{2}$

$= 5\sqrt{2}$

(b) $\dfrac{22}{4-\sqrt{5}} = \dfrac{22(4+\sqrt{5})}{(4-\sqrt{5})(4+\sqrt{5})}$

$= \dfrac{22(4+\sqrt{5})}{16+\cancel{4\sqrt{5}}-\cancel{4\sqrt{5}}-5}$

$= \dfrac{22(4+\sqrt{5})}{11}$

$= 2(4+\sqrt{5})$

$= 8+2\sqrt{5}$

Question 2

(a) $f(x)=3x^2+12x+8$

$\Rightarrow f(x)=3\left[x^2+4x+\frac{8}{3}\right]$

$\Rightarrow f(x)=3\left[(x+2)^2-4+\frac{8}{3}\right]$

$\Rightarrow f(x)=3(x+2)^2-12+8$

$\Rightarrow f(x)=3(x+2)^2-4$

(b) $f(x)_{MIN}=-4$

(OCCURS WHEN $x=-2$)

(c) $f(x)=0$

$\Rightarrow 3x^2+12x+8=0$

$\Rightarrow 3(x+2)^2-4=0$

$\Rightarrow 3(x+2)^2=4$

$\Rightarrow (x+2)^2=\frac{4}{3}$

$\Rightarrow x+2=\pm\sqrt{\frac{4}{3}}$

$\Rightarrow x+2=\pm\frac{2}{\sqrt{3}}$

$\Rightarrow x+2=\pm\frac{2}{3}\sqrt{3}$

$\Rightarrow x=-2\pm\frac{2}{3}\sqrt{3}$

Question 3

(a) $f(x)=x^3-3x^2-4x+12$

$f(3)=27-27-12+12=0$

$\therefore (x-3)$ IS A FACTOR

(b) BY LONG DIVISION OR GROUPING

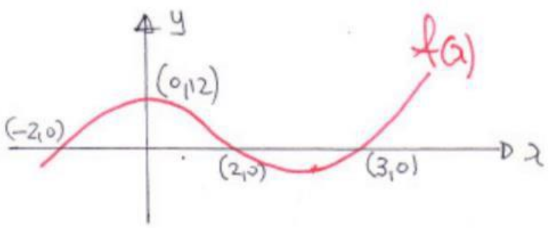
$f(x)=x^3-3x^2-4x+12$

$f(x)=x^2(x-3)-4(x-3)$

$f(x)=(x-3)(x^2-4)$

$f(x)=(x-3)(x-2)(x+2)$

(c)

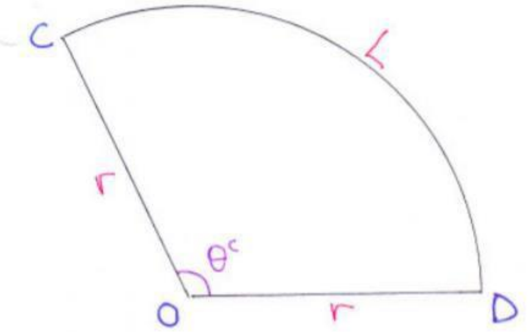


$+x^3 \Rightarrow$ ∼

$x=0 \Rightarrow y=12$

$y=0 \Rightarrow x= -2,\ 2,\ 3$

Question 4



- $A = 0.25$

$\frac{1}{2}r^2\theta^c = \frac{1}{4}$

$r^2\theta = \frac{1}{2}$

$\boxed{2r^2\theta = 1}$

- $P = 2$

$2r + L = 2$

$2r + r\theta = 2$

$\boxed{r\theta = 2 - 2r}$

$\Rightarrow 2r(r\theta) = 1$

$2r(2-2r) = 1$

$4r - 4r^2 = 1$

$0 = 4r^2 - 4r + 1$

$0 = (2r-1)^2$

$r = \frac{1}{2} = 0.5m$

USING $r\theta = 2 - 2r$

$\frac{1}{2}\theta = 1$

$\theta = 2^c$

Question 5

a) $6^{3x+2} = 30$

$\log_6 30 = 3x + 2$

$x = \frac{(\log_6 30) - 2}{3}$

$x = -0.0339$

b) $\log_4(12y+5) - \log_4(1-y) = 2$

$\Rightarrow \log_4\left(\frac{12y+5}{1-y}\right) = 2$

$\Rightarrow \log_4\left(\frac{12y+5}{1-y}\right) = 2\log_4 4$

$\Rightarrow \log_4\left(\frac{12y+5}{1-y}\right) = \log_4 16$

$\Rightarrow \frac{12y+5}{1-y} = 16$

$\Rightarrow 12y + 5 = 16 - 16y$

$\Rightarrow 28y = 11$

$\Rightarrow y = \frac{11}{28}$

$\simeq 0.393$

(c) $8^{2t} - 8^t - 6 = 0$

$\Rightarrow (8^t)^2 - (8^t) - 6 = 0$

LET $a = 8^t$

$\Rightarrow a^2 - a - 6 = 0$

$\Rightarrow (a-3)(a+2) = 0$

$\Rightarrow a = 3$ or -2

$8^t = 3$ or -2

$8^t = 3$

$\log_8 3 = t$

$t = 0.528$

$8^t \neq -2$

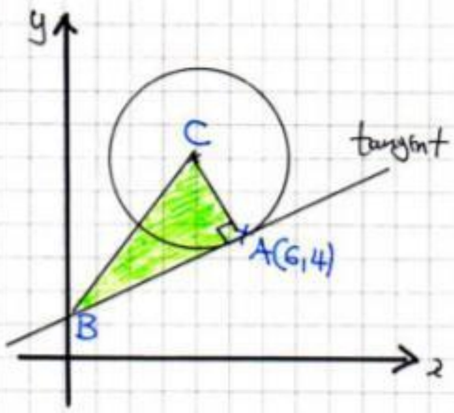
$8^t > 0$ for all t

Question 6

a) REWRITING THE EQUATION OF THE CIRCLE, TO READ THE CENTRE

$$x^2+y^2-10x-12y+56=0$$
$$x^2-10x+y^2-12y+56=0$$
$$(x-5)^2-25+(y-6)^2-36+56=0$$
$$(x-5)^2+(y-6)^2=5$$

$\therefore\ C(5,6)$ & $r=\sqrt{5}$



FIND THE GRADIENT OF AC, WHERE C(5,6) & A(6,4)

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{4-6}{6-5}=\frac{-2}{1}=-2$$

USING PERPENDICULAR GRADIENT OF $+\frac{1}{2}$ WE "OBTAIN THE TANGENT"

$$y-y_0=m(x-x_0)$$
$$y-4=\tfrac{1}{2}(x-6)$$
$$2y-8=x-6$$
$$2y=x+2 \quad \text{or} \quad y=\tfrac{1}{2}x+1$$

b) FIND THE CO-ORDINATES OF B

when $x=0$ $\quad 2y=2$

$y=1$

$\therefore B(0,1)$

I would say just draw a RA triangle, reduces the likelihood of mistakes with negatives!

FIND THE DISTANCE AB, WHERE A(6,4) & B(0,1)

$$\Rightarrow d=\sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$$
$$\Rightarrow |AB|=\sqrt{(1-4)^2+(0-6)^2}$$
$$\Rightarrow |AB|=\sqrt{9+36}$$
$$\Rightarrow |AB|=\sqrt{45}=3\sqrt{5}$$

HENCE THE REQUIRED AREA IS GIVEN BY

$$\Rightarrow \text{AREA}=\tfrac{1}{2}|AB||AC|$$
$$\Rightarrow \text{AREA}=\tfrac{1}{2}\times 3\sqrt{5}\times\sqrt{5}$$
$$\Rightarrow \text{AREA}=\tfrac{15}{2}$$

Question 7

$$4\tan^2\theta\cos\theta = 15$$

$$\Rightarrow 4\left(\frac{\sin\theta}{\cos\theta}\right)^2\cos\theta = 15$$

$$\Rightarrow \frac{4\sin^2\theta}{\cos^2\theta} \times \cos\theta = 15$$

$$\Rightarrow \frac{4\sin^2\theta}{\cos\theta} = 15$$

$$\Rightarrow 4\sin^2\theta = 15\cos\theta$$

$$\Rightarrow 4(1-\cos^2\theta) = 15\cos\theta$$

$$\Rightarrow 4 - 4\cos^2\theta = 15\cos\theta$$

$$\Rightarrow 0 = 4\cos^2\theta + 15\cos\theta - 4$$

$$\Rightarrow (4\cos\theta - 1)(\cos\theta + 4) = 0$$

$$\Rightarrow \cos\theta = \frac{1}{4}, \cancel{-4}$$

$$\arccos\left(\tfrac{1}{4}\right) = 75.5^\circ$$

$$\theta = 75.5^\circ \pm 360n$$
$$\theta = 284.5 \pm 360n \qquad n = 0, 1, 2, 3 \ldots$$

$$\theta_1 = 75.5^\circ$$
$$\theta_2 = 284.5^\circ$$

Question 8

$$1 + 2\sin(\theta + 25^\circ) = 2.532, \qquad 25^\circ \leq \theta + 25^\circ < 385^\circ$$

360° range - 2 solutions

$$2\sin(\theta + 25^\circ) = 1.532$$

$$\sin(\theta + 25^\circ) = 0.766$$

$$\theta + 25^\circ = 49.996^\circ \qquad \sin^{-1}(0.766)$$

$$\theta + 25^\circ = 50^\circ, \; 130^\circ \qquad (180 - x)$$

$$\theta = 25^\circ, \; 105^\circ$$

All answers for $\theta + 25^\circ$ and θ needed!

Question 9

$$f(x) \longmapsto f\left(\tfrac{1}{3}x\right)$$

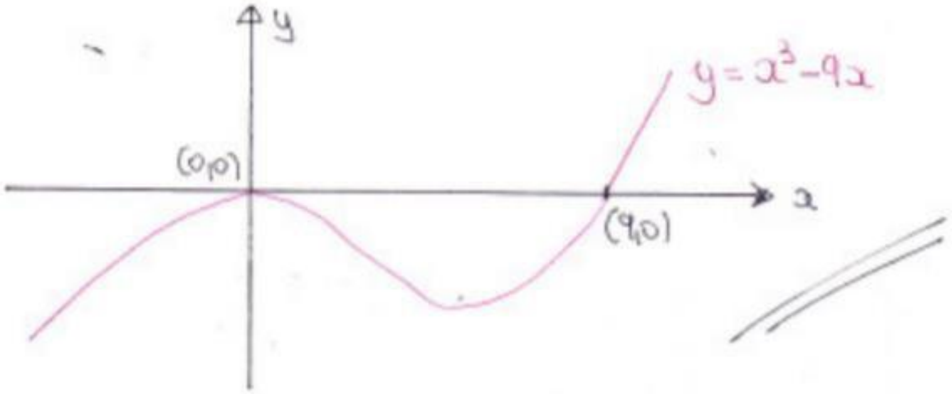
$$\sqrt{27x^3 + 1} \qquad \sqrt{27\left(\tfrac{1}{3}x\right)^3 + 1}$$

$$\sqrt{x^3 + 1}$$

Question 10

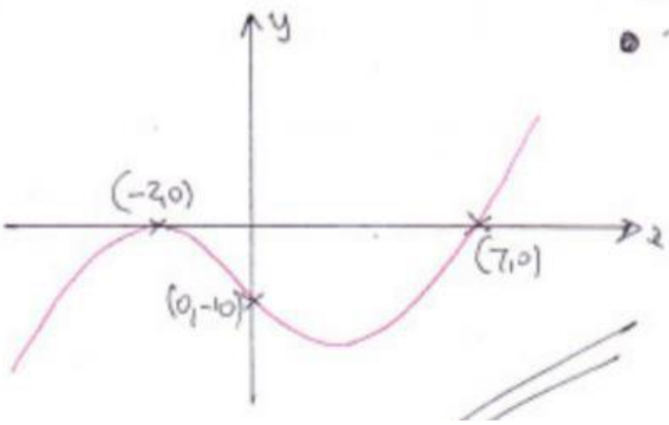
(a) $y = x^3 - 9x = x^2(x-9)$

- 
- $x=0, y=0 \Rightarrow (0,0)$
- $y=0, x = \begin{cases} 0 \\ 9 \end{cases} \Rightarrow$ $(0,0) \leftarrow$ TOUCHING POINT
  $(9,0) \leftarrow$ CROSSING POINT



(b) $y = (x+2)^3 - 9(x+2)$

- $x \longmapsto (x+2)$
  ∴ TRANSLATION, "TO THE LEFT", 2 UNITS
- TO FIND y INTERCEPT, $x=0$
  $y = 2^3 - 9\times 2 = -10$
  ∴ $(0,-10)$



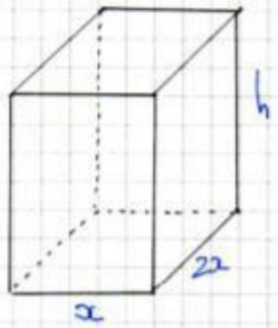
Question 11

(a) $\cos(x) \longmapsto \cos(2x) \longmapsto -\cos 2x \longmapsto -\cos 2x + 3$

STRETCH, HORIZONTALLY SCALE FACTOR $\frac{1}{2}$ — REFLECTION IN THE x AXIS — TRANSLATION, UPWARDS BY 3 UNITS

Question 12

a)



CONSTRAINT $V = 1000\,\text{cm}^3$

$\Longrightarrow V = x(2x)h$

$\Longrightarrow 1000 = 2x^2h$

$\Longrightarrow x^2h = 500$

$xh = \frac{500}{x}$

$\boxed{6xh = \frac{3000}{x}}$

SURFACE AREA ($A\,\text{cm}^2$)

$\Longrightarrow A = 2\left[2x^2 + xh + 2xh\right]$

$\Longrightarrow A = 4x^2 + 6xh$

$\Longrightarrow A = 4x^2 + \frac{3000}{x}$

AS REQUIRED

b) $A = 4x^2 + 3000x^{-1}$

$\Longrightarrow \frac{dA}{dx} = 8x - 3000x^{-2}$

FOR STATIONARY VALUES $\frac{dA}{dx} = 0$

$\Longrightarrow 8x - \frac{3000}{x^2} = 0$

$\Longrightarrow 8x = \frac{3000}{x^2}$

$\Longrightarrow 8x^3 = 3000$

$\Longrightarrow x^3 = 375$

$\Longrightarrow x = \sqrt[3]{375} \approx 7.21\,\text{cm}$

c) $A = 4x^2 + \frac{3000}{x}$

$\Longrightarrow A_{MIN} = 4(7.21...)^2 + \frac{3000}{7.21...}$

$\Longrightarrow A_{MIN} \approx 624\,\text{cm}^2$

TO JUSTIFY IT IS A MIN, USE 2$^{\text{ND}}$ DERIVATIVE

$$\Rightarrow \frac{dA}{dx} = 8x - 3000x^{-2}$$

$$\Rightarrow \frac{d^2A}{dx^2} = 8 + 6000x^{-3}$$

$$\Rightarrow \frac{d^2A}{dx^2} = 8 + \frac{6000}{x^3}$$

$$\Rightarrow \left.\frac{d^2A}{dx^2}\right|_{x=7.21\ldots} = 8 + \frac{6000}{(7.21\ldots)^3} = 24 > 0$$

INDEED A MINIMUM

Question 13

THE DERIVATIVE IS FORMALLY GIVEN BY

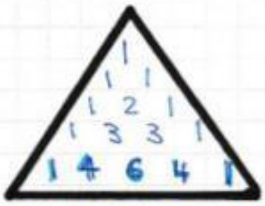
$$f'(x) = \lim_{h \to 0}\left[\frac{f(x+h) - f(x)}{h}\right]$$

IN THIS CASE WE HAVE

$$f(x) = x^4$$
$$f(x+h) = (x+h)^4$$

EXPANDING BINOMIALLY WE HAVE

$$(x+h)^4 = 1x^4h^0 + 4x^3h^1 + 6x^2h^2 + 4x^1h^3 + 1x^0h^4$$
$$(x+h)^4 = x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4$$



TIDYING UP NEXT

$$f(x+h) - f(x) = (x+h)^4 - x^4 = (\cancel{x^4} + 4x^3h + 6x^2h^2 + 4xh^3 + h^4) - \cancel{x^4}$$

FINALLY WE HAVE

$$f'(x) = \lim_{h \to 0}\left[\frac{f(x+h) - f(x)}{h}\right] = \lim_{h \to 0}\left[\frac{4x^3h + 6x^2h^2 + 4xh^3 + h^4}{h}\right]$$

$$= \lim_{h \to 0}\left[4x^3 + \cancel{6x^2h} + \cancel{4xh^2} + \cancel{h^3}\right]$$

$$= 4x^3$$

Question 14

(a) IF $f'(x) = 3x^2 - 8x + 4$

THEN

$f(x) = \int 3x^2 - 8x + 4 \, dx$

$f(x) = x^3 - 4x^2 + 4x + C$

BUT CUBIC GOES THROUGH $(0,0)$

$0 = 0 - 0 + 0 + C$

$\therefore C = 0$

$\therefore f(x) = x^3 - 4x^2 + 4x$

(b) $f(x) = x^3 - 4x^2 + 4x$

$f(x) = x(x^2 - 4x + 4)$

$f(x) = x(x-2)^2$

$\therefore$ when $y = 0$

$x = 0$ or 2

$\therefore P(2,0)$

(c)

$$\int_0^2 x^3 - 4x^2 + 4x \, dx$$

$$\left[\frac{x^4}{4} - \frac{4x^3}{3} + 2x^2\right]_0^2$$

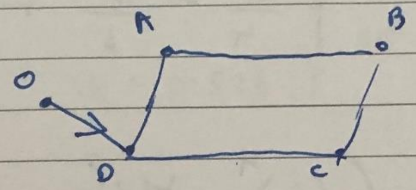
$$\left(\frac{(2)^4}{4} - \frac{4(2)^3}{3} + 2(2)^2\right) - (0)$$

$$\frac{4}{3}$$

Question 15

(a) $\vec{OA} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$ $\vec{OB} = \begin{pmatrix} 5 \\ -3 \\ 4 \end{pmatrix}$ $\vec{OC} = \begin{pmatrix} 0 \\ 7 \\ -4 \end{pmatrix}$

Parallelogram diagram



$\vec{OD} = \vec{OA} + \vec{AD}$

(other options too)

$\vec{OD} = \vec{OA} + \vec{BC}$ parallelogram so $\vec{AD} = \vec{BC}$

$\vec{OD} = \vec{OA} + \underbrace{\vec{BO}}_{[-\vec{OB}]} + \vec{OC}$

$\vec{OD} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + \begin{pmatrix} -5 \\ 3 \\ -4 \end{pmatrix} + \begin{pmatrix} 0 \\ 7 \\ -4 \end{pmatrix}$ $\boxed{\vec{OD} = \begin{pmatrix} -3 \\ 13 \\ -9 \end{pmatrix}}$

1b) Distance AC, need vector $\vec{AC}$

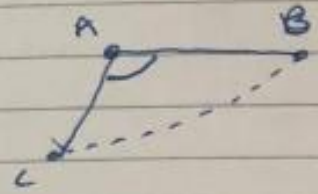
$\vec{AC} = \vec{AO} + \vec{OC}$

$\vec{AC} = \begin{pmatrix} -2 \\ -3 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 7 \\ -4 \end{pmatrix}$

$\vec{AC} = \begin{pmatrix} -2 \\ 4 \\ -3 \end{pmatrix}$ $\therefore$ $|\vec{AC}|$ or $AC = \sqrt{2^2 + 4^2 + 3^2}$

$|\vec{AC}|$ or $AC = \sqrt{29}$

Angle BAC will required cosine rule with 3 sides



$|\vec{AC}| = \sqrt{29}$

$\vec{BC} = \begin{pmatrix} -5 \\ 10 \\ -8 \end{pmatrix}$ Used workings from (a)

$\therefore |\vec{BC}| = \sqrt{5^2 + 10^2 + 8^2}$

$|\vec{BC}| = \sqrt{189}$

lastly $\vec{AB} = \vec{AO} + \vec{OB}$

$\vec{AB} = \begin{pmatrix} -2 \\ -3 \\ 1 \end{pmatrix} + \begin{pmatrix} 5 \\ -3 \\ 4 \end{pmatrix}$ $\vec{AB} = \begin{pmatrix} 3 \\ -6 \\ 5 \end{pmatrix}$

$\therefore |\vec{AB}| = \sqrt{3^2 + 6^2 + 5^2}$

$|\vec{AB}| = \sqrt{70}$

Now

$BC^2 = AC^2 + AB^2 - 2 \times AC \times AB \cos A$

$189 = 29 + 70 - 2 \times \sqrt{29} \times \sqrt{70} \cos A$

$\cos A = \dfrac{70 + 29 - 189}{2\sqrt{29}\sqrt{70}}$

$A = 177.2^\circ$ (odd!)

Question 16

(a) $\boxed{S_n = \frac{n}{2}\left[2a+(n-1)d\right]}$

$\Rightarrow 1070 = \frac{20}{2}\left[2a+19d\right]$

$\Rightarrow 1070 = 10\left[2a+19d\right]$

$\Rightarrow \boxed{107 = 2a+19d}$

$u_5 + u_{10} = 65$

$(a+4d)+(a+9d) = 65$

$\boxed{2a + 13d = 65}$

$2a = 107 - 19d$

$2a = 65 - 13d$

$107 - 19d = 65 - 13d$

$42 = 6d$

$d = 7$

& $2a = 65 - 13d$

$2a = 65 - 13\times 7$

$2a = 65 - 91$

$2a = -26$

$a = -13$

(b) $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$

$\Rightarrow S_{30} = \frac{30}{2}\left[2\times(-13)+29\times 7\right]$

$\Rightarrow S_{30} = 15\left[-26+203\right]$

$\Rightarrow S_{30} = 15\times 177$

$\Rightarrow S_{30} = 2655$

$$\begin{array}{r} 1770 \\ 885 \\ \hline 2655 \end{array}$$

Question 17

$u_2 = 4$

$S_\infty = 18$

(a) • $u_n = ar^{n-1}$

$\boxed{4 = ar}$

• $S_\infty = \frac{a}{1-r}$

$18 = \frac{a}{1-r}$

$\boxed{18 - 18r = a}$

$4 = (18-18r)r$

$4 = 18r - 18r^2$

$18r^2 - 18r + 4 = 0$

$9r^2 - 9r + 2 = 0$ AS REQUIRED

(b) $(3r-1)(3r-2) = 0$

$r = \frac{1}{3}, \frac{2}{3}$

$\boxed{\text{NOW } a = \frac{4}{r}}$

$\therefore a = \frac{4}{1/3} = 12, \quad \frac{4}{2/3} = 6$

$\therefore a = 12$ with $r = \frac{1}{3}$

OR

$a = 6$ with $r = \frac{2}{3}$

(c) $S_n = \dfrac{a(1-r^n)}{1-r}$

$r = \frac{2}{3}$, $a = 6$

$S_n > 17.975$ easier ~~to~~ solve $=$ first

$17.975 = \dfrac{6\left(1-\left(\frac{2}{3}\right)^n\right)}{1-\frac{2}{3}}$ don't expand bracket...

$17.975 = 18\left(1-\left(\frac{2}{3}\right)^n\right)$

$0.9986\dot{1} = 1-\left(\frac{2}{3}\right)^n$

$\left(\frac{2}{3}\right)^n = 0.0013\dot{8}$

$\log_{\frac{2}{3}} 0.0013\dot{8} = n$

$n = 16.226$

$\underline{n = 17}$

question says exceed, so $n = 17$.

check:

$S_{16} = \dfrac{6\left(1-\left(\frac{2}{3}\right)^{16}\right)}{1-\frac{2}{3}} = 17.9725$

$S_{17} = \dfrac{6\left(1-\left(\frac{2}{3}\right)^{17}\right)}{1-\frac{2}{3}} = 17.98$

Question 18

(a) $y = \sqrt{4-12x} = (4-12x)^{\frac{1}{2}} = 4^{\frac{1}{2}}(1-3x)^{\frac{1}{2}} = 2(1-3x)^{\frac{1}{2}}$

$= 2\left[1 + \frac{1}{2}(-3x) + \frac{\frac{1}{2}(-\frac{1}{2})}{1\times 2}(-3x)^2 + \frac{\frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})}{1\times 2\times 3}(-3x)^3 + O(x^4)\right]$

$= 2\left[1 - \frac{3}{2}x - \frac{9}{8}x^2 - \frac{27}{16}x^3 + O(x^4)\right]$

$= 2 - 3x - \frac{9}{4}x^2 - \frac{27}{8}x^3 + O(x^4)$

(b) $(12x-4)(4-12x)^{\frac{1}{2}}$

$(12x-4)\left(2 - 3x - \frac{9}{4}x^2 - \frac{27}{8}x^3 + O(x^4)\right)$

$-36x^2$

$9x^2$

$\therefore -27x^2$

$\therefore -27$

Question 19

$$\frac{2x^3 + x^2 - 4x + 1}{x^2 + x - 2}$$

	$2x$	-1	R
x^2	$2x^3$	$-x^2$	$+x$
$+x$	$+2x^2$	$-x$	-1
-2	$-4x$	$+2$	

lots of other methods are possible for polynomial division, all giving the same answer.

Also a quick check on my graphical calculator shows $(x-1)$ could be cancelled to begin...

so given $2x - 1 + \dfrac{x-1}{x^2 + x - 2}$

$$2x - 1 + \frac{(x-1)}{(x-1)(x+2)}$$

$$2x - 1 + \frac{1}{x+2}$$

$$\frac{2x^3 + x^2 - 4x + 1}{(x-1)(x+2)} \equiv Ax + B + \frac{C}{x+D}$$

Even if you spot this factorisation, unless you 'guess' what D is, a partial fraction style method is difficult!

Task 2 Solutions can be found with questions

Task 3 solutions

Kinematics – constant acceleration

Standard

2	(i)	Acceleration is $0.09\,ms^{-2}$	B1	[1]	
	(ii)	$[D = \frac{1}{2}(8+4)0.18$ or $D = (0 + \frac{1}{2}0.09 \times 2^2) + (0.18 \times 4 + \frac{1}{2}0 \times 4^2) + (0.18 \times 2 - \frac{1}{2}0.09 \times 2^2)]$	M1		For using the idea that area represents distance or for repeated use of $s = ut + \frac{1}{2}at^2$
		Distance is 1.08 m	A1	[2]	
	(iii)	$[\frac{1}{2}3V = 1.08]$	M1		For using area of triangle = area of trapezium
		Greatest speed is $0.72\,ms^{-1}$	A1	[2]	

Challenging

6	(i)	$\frac{1}{2}5v_{max} = \pm 10$	M1		For using the idea that the area of the relevant triangle represents distance
		Greatest speed is $4\,ms^{-1}$	A1	2	
	(ii)	$V/3 = 2$ or $V = 0 + 2\times3$	M1		For using the idea that the gradient represents acceleration or $v = 0 + at$
		$V = 6$	A1	2	
	(iii)		M1		For an attempt to find the area of the trapezium in terms of T (or of t) and equate with 34.5
		$\frac{1}{2}(T + 9.5)6 = 34.5$ or $\frac{1}{2}(t - 18 + 9.5)6 = 34.5$	A1 ft		Any correct form of equation in T (ot t)
		Time is 2 s	A1	3	
	(iv)	$d = \dfrac{6}{24.5 - (18 + 2)}$	M1		For using the idea that minus the gradient represents deceleration
		Deceleration is $4/3\,ms^{-2}$	A1ft	2	

Hard

3	(i)		M1		For using $0 = u^2 - 2gs$
		$u^2 = 2 \times 10 \times 45$; speed is $30ms^{-1}$	A1	[2]	
	(ii)	$[40 = 30t - 5t^2 \rightarrow t = 2, 4]$			For using $s = ut - \frac{1}{2}gt^2$ with $s = 40$, $u = 30$ and $T = t_2 - t_1$ or $s = ut + \frac{1}{2}gt^2$ $s = 5$, $u = 0$ and $T = 2t$
		$[5 = \frac{1}{2}10t^2 \rightarrow t = 1]$	M1		
		Time above the ground is 2 s	A1ft	[2]	
Special Ruling for candidates who assume, without justification, that the length of time required is that of the upward movement only. (maximum mark 1).					
	(ii)	$5 = \frac{1}{2}10t^2 \rightarrow t = 1$, the length of time required is 1 s	B1	B1	
	(iii)	Max. height above top of cliff $= \frac{1}{2}g(17 \div 4)$ $(= 21.25)$	B1		
		$[0 = V^2 - 2g(40 + 21.25)$	M1		For using $0 = u^2 - 2gs$
		Speed is $35\,ms^{-1}$	A1	[3]	

Kinematics - variable acceleration

Standard

5	(i)		M1	For attempting to use $x(t) = \int v dt$
		$x = 0.01t^3 \quad (+C)$	A1	
		$2.5 = 0.01 \times 5^3 + C$	DM1	For substituting $x = 2.5$ and $t = 5$ and attempting to find C
		$x = 0.01t^3 + 1.25$	A1 ft 4	ft candidate's a where $x = at^3 + C$
	(ii)	$0.01t^3 + 1.25 = 11.25$	M1	For attempting to solve $x(t) = 11.25$ (equation needs to be of the form $at^3 = b$)
		$t = 10$	A1	
		Velocity is 3ms^{-1}	B1ft 3	ft for value of $0.03t^2$

Challenging

7	(i)	$(dv/dt) = -0.02t + 0.5$ **or** $v = -0.01[(t - T)^2 - 100V]$ where $T = 25$ and $V = 5.25$ (or equivalent)	B1		
			M1		For solving $dv/dt = 0$ or for selecting $t = T$ or $v_{max} = V$ May be implied when $v_{max} = V$ is selected and T is 25 in the 'B1' expression for v
		$t = 25$	A1		
		Maximum velocity is 5.25ms^{-1}	A1	[4]	
	(ii)		M1		For integrating v(t)
		$s_2 = -0.01t^3/3 + 0.5t^2/2 - t$	A1		
			M1		For using limits 10 and 30
		$s_2 = (-90 + 225 - 30) - (-10/3 + 25 - 10)$ $(= 93.3\text{m})$	A1		
			M1		For evaluating v(10) and v(30)
		$v(10) = 3$ and $v(30) = 5$	A1		
			M1		For evaluating s_1 and s_3
		$s_1 = ½\ 3\text{x}10$ and $s_3 = ½\ 5\text{x}50$	A1ft		ft incorrect values of v(10) and/or v(30)
		Distance is 233m	A1ft	[9]	ft $140 + s_2$ (depends on the 1st M1)
					SR for candidates who treat the first line segment as part of the curve in part **(ii)** (max. mark 6/9) Integration — M1 A1 as scheme $s_1 + s_2 = 105$ — A1 $v(30) = 5$ — B1 $s_3 = ½\ 5\text{x}50$ — B1ft Distance is 230m — A1ft (ft $125 + s1 + s_2$)

Hard

7	(i)		M1	For integrating v_1 to find s_1
		$\int_0^{15} v_1 dt = 225 \rightarrow$	A1	
		$A[(15^2/2 - 0.05 \times 15^3/3) - (0 - 0)] = 225$		
		$A = 4$	A1	
		$[4(15 - 0.05 \times 15^2) = B/15^2]$	M1	For using $v_1(15) = v_2(15)$
		$B = 3375$	A1	AG
			[5]	
	(ii)	$s_2(t) = Bt^{-1}/(-1)$ (+ C)	B1	
		$[-3375/15 + C = 225]$	M1	For using $s_2(15) = 225$ to find C
		Distance travelled is $[450 - 3375/t]$ m (for $t \geqslant 15$)	A1	
			[3]	
	(iii)	$[450 - 3375/t = 315]$	M1	For attempting to solve $s_2(t) = 315$
		$[v = 3375/25^2]$	M1	For substituting into $v = 3375/t^2$
		Speed is 5.4 ms^{-1}	A1	
			[3]	
Alternative for **7(ii)**				
		$s = \int_{15}^{t} 3375t^{-2} dt = -3375(\frac{1}{t} - \frac{1}{15})$	B1	
		$= 225 - 3375/t$		
		Distance travelled $= 225 + (225 - 3375/t)$	M1	
		Distance travelled is $[450 - 3375/t]$ m (for $t \geqslant 15$)	A1	

<u>Forces; Non – slopes</u>

Question 1

3			**M1**		**For resolving forces in the 'j' direction**
		$F\sin 50° = F\sin 20° + 12$	**A1**		
		$F = 28.3$	**A1**		
			M1		**For resolving forces in the 'i' direction**
		$G = F\cos 50° + F\cos 20°$	**A1**		
		$G = 44.8$	**A1 ft**	**6**	**Ft value of 1.5825*F***

Question 2

4	**(i)**		M1		For applying Newton's second law to P or to Q (3 terms)
		$0.6\,g - T = 0.6a$	A1		
		$T - 0.2\,g = 0.2a$	A1		
					Allow B1 for $0.6\,g - 0.2\,g = (0.6 + 0.2)a$ as an alternative for either of the above A marks
		Acceleration is 5 ms^{-2}	B1		
		Tension is 3 N	A1	5	
	(ii)	$[0.9 = ½\, 5t^2]$	M1		For using $s = ut + ½\, at^2$
		Time taken is 0.6 s	A1ft	2	ft $\sqrt{1.8/a}$

Question 3

(a) Car + trailer: $2100a = 2380 - 280 - 630$	M1 A1
$= 1470 \Rightarrow a = \underline{0.7 \text{ m s}^{-2}}$	A1 (3)
(b) e.g. trailer: $700 \times 0.7 = T - 280$	M1 A1√
$\Rightarrow T = \underline{770 \text{ N}}$	A1 (3)
(c) Car: $1400a' = 2380 - 630$	M1 A1
$\Rightarrow a' = 1.25 \text{ m s}^{-2}$	A1
distance = $12 \times 4 + ½ \times 1.25 \times 4^2$	M1 A1√
$= \underline{58 \text{ m}}$	A1 (6)
(d) Same acceleration for car and trailer	B1 (1)

Question 4

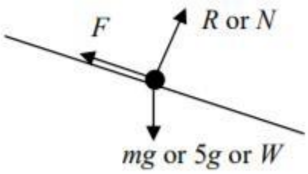
7	**(i)**	$T\cos60° = 75\cos30° \to T = 130$	B1		Accept $75\sqrt{3}$
			M1		For resolving forces vertically (4 terms)
		$T\sin60° + 75\sin30° + R = 20g$	A1ft		ft consistent sin/cos mix
		$[130\sin60° + 75\sin30° + R = 200]$	M1		For substituting for T and solving for R
		Magnitude is 50 N	A1	5	Accept 49.9
	(ii)		M1		For resolving forces horizontally
		$T\cos60° + 25 = 75\cos30°$ ($T = 79.9$)	A1ft		ft consistent sin/cos mix ($T = 14.4$)
		$[79.9\sin60° + 75\sin30° + R = 200]$	M1		For resolving forces vertically (4 terms) and substituting for T
		$R = 93.3$	A1		May be implied by final answer
		$[\mu = 25/93.3]$	M1		For using $\mu = 25/R$
		Coefficient is 0.268 ($= 2 - \sqrt{3}$)	A1ft	6	ft for μ = value obtained from 25/candidate's R, including her/his answer in **(i)** but excluding $R = 20$ g

Question 5

6	(i)		M1		For applying Newton's second law to *A* or to *B* or using (M + m)a = Mg – F
		$0.45a = 0.45g - T$ and $0.2a = T - F$ or $(0.45 + 0.2)a = 0.45g - F$	A1		
		$F = 0.3 \times 0.2g$	B1		
			M1		For substituting for F and solving for a
		Acceleration is $6\ \text{ms}^{-2}$	A1		
		$[v^2 = 2 \times 6 \times [2 - (2.8 - 2.1)]$	M1		For using $v^2 = (0^2) + 2as$ (s must be less than 2)
		Speed is $3.95\ \text{ms}^{-1}$	A1	[7]	AG
	(ii)	$0.2a_2 = -0.06g$	B1ft		ft incorrect F
			M1		For using $v^2 = 3.95^2 + 2a_2[2.1 - \text{distance moved by B}]$
		$v^2 = 15.6 + 2(-3)(0.8)$	A1		
		Speed is $3.29\ \text{ms}^{-1}$	A1	[4]	

<u>Forces - Slopes</u>

Question 1

6(a)		B1	1	Correct force diagram with labels and arrows Accept components of the weight if shown in a different notation with the weight also shown. B0 if components are shown instead of the weight.
(b)	$(R =)5\times9.8\cos40° = 37.5$ N **AG**	M1 A1	2	Attempt at resolving perpendicular to the slope (eg 49sin40°) Correct value from correct working
(c)	$5\times0.8 = 5\times9.8\sin40° - \mu\times5\times9.8\cos40°$	B1 M1 A1 A1		Use of $F = \mu R$ at any stage and with any F but with $R = 37.5$ OE Three term equation of motion from resolving parallel to the slope with weight component, friction and *ma* term. Correct terms seen (may be as 31.5, 37.5μ (or F) and 4) Correct signs
	$\mu = \dfrac{5\times9.8\sin40° - 5\times0.8}{5\times9.8\cos40°} = 0.733$	m1 A1	6	Solving for μ A1: Correct value for μ Allow 0.732 but not $\frac{11}{15}$ unless converted to a decimal
(d)	There is less friction so the coefficient of friction must be less.	B1 B1	2	Less friction Smaller coefficient of friction If the answer and explanation contradict each other, award no marks
	Total		**11**	

Question 2

6	**(i)**		M1		For using Newton's 2nd law for P or for Q; or for using (M – m)g × 0.8 = (M + m)a
		$0.6g \times 0.8 - T = 0.6a$ and $T - 0.4g \times 0.8 = 0.4a$ or $(0.6 - 0.4)g \times 0.8 = (0.6 + 0.4)a$	A1		
			M1		For solving for T or for a
		Tension is 3.84 N or acceleration is 1.6ms^{-2}	A1		
		Acceleration is 1.6 ms^{-2} or tension is 3.84 N	A1	[5]	
	(ii)	$2 = 1.6t_1$ ($t_1 = 1.25$)	B1ft		
			M1		For using $0 + u + at$ with $a = -0.8g$
		$0 = 2 - 0.8gt_2$ ($t_2 = 0.25$)	A1		
		Time taken in 1.5 s	A1ft	[4]	ft incorrect acceleration in **(i)**

Question 3

5	**(i)**	R = 2.6 × (12 ÷ 13) (= 2.4)	B1		
		[F = 0.2 × 2.4]	M1		For using F = μR
		[T – 2.6(5 ÷ 13) – F = 0.26a, 5.4 – T = 0.54a]	M1		For applying Newton's 2nd law to A or to B.
		For any two of T – 1 – 0.48 = 0.26a, 5.4 – T = 0.54a or (5.4 – 1 – 0.48) = (0.54 + 0.26)a	A1		
		Acceleration is 4.9 ms^{-2}	B1		
		Tension is 2.75 N (2.754 exact)	A1	[6]	
	(ii)	[$s = \frac{1}{2}\, 4.9 \times 0.4^2$]	M1		For using $s = \frac{1}{2}at^2$
		Distance is 0.392 m	A1	[2]	