newcollege Pontefract

Mathematics

Y11 to Y12 Further Mathematics Summer Independent Learning

June to August 2022

Important notes:

1. **This is your Maths SIL for both Maths and Further Maths (Please do not do that regular Maths SIL)**
2. **This means there is (roughly) twice as much as your other two subjects, as it's two sets of lessons**
3. **You do not need to do all the questions in section 1**

There are two mandatory tasks and one strongly recommended task to work through.
Please read the following instructions very carefully and ensure you label and collate all your work ready for checking in September.

For your first maths lesson please bring

- A large A4 folder with five subject dividers.
- These instructions with the tables filled in (print out/copy the tables onto A4 paper).
- Dated and titled work done on each of the topics listed in Task 1 & 3.
- The two practice initial tests (Task 2), fully marked and reviewed.
- A list of questions you need to ask prior to doing your initial test.

Task 1: Preparation Work (Mandatory)

1. For each topic, work through video.
2. Complete worksheet **using the technique and layout** used in the video.
3. Make sure you title and date your work. **[You do not need to complete all questions in each section]**
4. Mark and correct work.
5. Do improvement work as necessary.
6. Repeat for each topic.
7. Keep track by filling in the following table.
8. Collate your work for each topic together so it is easy to check in September. *(See point 3!)*

Topic	Video(s) *(Tick)*	Worksheet *(Tick)*	Details of Improvement Work Completed
B1 Indices			
B2 Surds			
B3 Quadratics			
B4 Simultaneous Equations			
B5 Inequalities			
E1 Triangle Geometry			

Task 2 (Mandatory)

1. Do Practice Initial Test 1 under exam conditions.
2. Mark and correct your test and identify any improvement work necessary.
3. Fill in the review sheet below.
4. Revisit relevant videos and worksheets.
5. Update review sheet with details of work completed.

Topic	**Score**	**Improvement Work to Do**	**Tick**
B1 Indices	11		
B2 Surds	10		
B3 Quadratics	49		
B4 Simultaneous Equations	11		
B5 Inequalities	11		
E1 Triangle Geometry	12		
Total	114		

6. Do Practice Initial Test 2 under exam conditions.
7. Mark and correct your test and identify any improvement work necessary.
8. Fill in the review sheet below.
9. Revisit relevant videos and worksheets.
10. Update review sheet with details of work completed.
11. Make a list of questions you need to ask prior to doing your initial test for real!

Topic	**Score**	**Improvement Work to Do**	**Tick**	**Questions to ask…**
B1 Indices	11			
B2 Surds	10			
B3 Quadratics	49			
B4 Simultaneous Equations	11			
B5 Inequalities	11			
E1 Triangle Geometry	12			
Total	114			

Task 3 :Additional Further Mathematics Preparation Work (Strongly recommend task)

1. For each topic section, work through the examples.
2. Complete the exercise, showing well-structured algebraic methods. You do not need to complete every question in the exercise, but should make sure you get enough practice with both the skills themselves, and with setting out your reasoning clearly and logically.
3. Mark and correct your work.

Topic	**Exercise** *(Tick)*	**Areas for Improvement**
2.1 Trigonometric Equations		
2.2 Other Trigonometric Methods		
3.1 Straight Line Graphs		
3.2 Basic Shape of Curved Graphs		
3.3 Factors		

Video hyperlinks

B1 Indices

https://youtu.be/1lThXgU08S0

https://youtu.be/v5bn4HZrmQs

https://youtu.be/W0h4rHj88ys

B2 Surds

https://youtu.be/jHelde32YtI

B3 Quadratics

https://youtu.be/Pziws8ojnlk

https://youtu.be/sn_joGVj15w

https://youtu.be/kk7p6hjn7hQ

https://youtu.be/toIqbX_NXHo

B4 Simultaneous Equations

https://youtu.be/4SRtwS5unwE

B5 Inequalities

https://youtu.be/wDut-In_7Wg

E1 Triangle Geometry

https://youtu.be/uVl6TAb0vBg

1.	Evaluate **a** 3^{-2} **b** $(\frac{2}{5})^0$ **c** $(-2)^{-6}$ **d** $(\frac{1}{6})^{-2}$ **e** $(1\frac{1}{2})^{-3}$ **f** $9^{\frac{1}{2}}$
2.	Work out **a** $4^{\frac{1}{2}} \times 27^{\frac{1}{3}}$ **b** $16^{\frac{1}{4}} + 25^{\frac{1}{2}}$ **c** $8^{-\frac{1}{3}} \div 36^{\frac{1}{2}}$ **d** $(-64)^{\frac{1}{3}} \times 9^{\frac{3}{2}}$ **e** $(\frac{1}{3})^{-2} - (-8)^{\frac{1}{3}}$ **f** $(\frac{1}{25})^{\frac{1}{2}} \times (\frac{1}{4})^{-2}$ **g** $81^{\frac{3}{4}} - (\frac{1}{49})^{-\frac{1}{2}}$ **h** $(\frac{1}{27})^{-\frac{1}{3}} \times (\frac{4}{9})^{-\frac{3}{2}}$
3.	Solve each equation. **a** $x^{\frac{1}{2}} = 6$ **b** $x^{\frac{1}{3}} = 5$ **c** $x^{-\frac{1}{2}} = 2$ **d** $x^{-\frac{1}{4}} = \frac{1}{3}$
4.	Express in the form x^k **a** $\sqrt{x}$ **b** $\frac{1}{\sqrt[3]{x}}$ **c** $x^2 \times \sqrt{x}$ **d** $\frac{\sqrt[4]{x}}{x}$
5.	Express each of the following in the form ax^b, where a and b are rational constants. **a** $\frac{4}{\sqrt{x}}$ **b** $\frac{1}{2x}$ **c** $\frac{3}{4x^3}$ **d** $\frac{1}{(3x)^2}$ **e** $\frac{2}{5\sqrt[3]{x}}$ **f** $\frac{1}{\sqrt{9x^3}}$
6.	Express in the form 2^k **a** 8^2 **b** $(\frac{1}{4})^{-2}$ **c** $(\frac{1}{2})^{\frac{1}{3}}$ **d** $16^{-\frac{1}{6}}$ **e** $8^{\frac{2}{5}}$ **f** $(\frac{1}{32})^{-3}$

Advanced Skills

1.	Express each of the following in the form 3^y, where y is a function of x. **a** 9^x **b** 81^{x+1} **c** $27^{\frac{x}{4}}$ **d** $(\frac{1}{3})^x$ **e** 9^{2x-1} **f** $(\frac{1}{27})^{x+2}$
2.	Given that $y = 2^x$, express each of the following in terms of y. **a** 2^{x+1} **b** 2^{x-2} **c** 2^{2x} **d** 8^x **e** 2^{4x+3} **f** $(\frac{1}{2})^{x-3}$
3.	Find the value of x such that **a** $2^x = 64$ **b** $5^{x-1} = 125$ **c** $3^{x+4} - 27 = 0$ **d** $8^x - 2 = 0$ **e** $3^{2x-1} = 9$ **f** $16 - 4^{3x-2} = 0$ **g** $9^{x-2} = 27$ **h** $8^{2x+1} = 16$
4.	Solve each equation. **a** $2^{x+3} = 4^x$ **b** $5^{3x} = 25^{x+1}$ **c** $9^{2x} = 3^{x-3}$ **d** $16^x = 4^{1-x}$ **e** $4^{x+2} = 8^x$ **f** $27^{2x} = 9^{3-x}$ **g** $6^{3x-1} = 36^{x+2}$ **h** $8^x = 16^{2x-1}$
5.	Solve the equation $25^x = 5^{4x+1}$.
6.	Given that $x = 2^{t-1}$ and $y = 2^{3t}$, **a** find expressions in terms of t for **i** xy **ii** $2y^2$ **b** Hence, or otherwise, find the value of t for which $2y^2 - xy = 0$.

Exam Questions *(OCR/MEI C1 Questions)*

1.	**Jan 05 Q5** Find the value of the following. (i) $\left(\frac{1}{3}\right)^{-2}$ [2] (ii) $16^{\frac{3}{4}}$ [2]
2.	**June 05 Q6** Simplify the following. (i) a^0 [1] (ii) $a^6 \div a^{-2}$ [1] (iii) $(9a^6b^2)^{-\frac{1}{2}}$ [3]
3.	**June 06 Q9** Simplify the following. (i) $\dfrac{16^{\frac{1}{2}}}{81^{\frac{3}{4}}}$ [2] (ii) $\dfrac{12(a^3b^2c)^4}{4a^2c^6}$ [3]
4.	**Jan 07 Q6** Find the value of each of the following, giving each answer as an integer or fraction as appropriate. (i) $25^{\frac{3}{2}}$ [2] (ii) $\left(\frac{7}{3}\right)^{-2}$ [2]
5.	**June 07 Q5** (i) Find a, given that $a^3 = 64x^{12}y^3$. [2] (ii) Find the value of $\left(\frac{1}{2}\right)^{-5}$. [2]

Answers – Basic Skills

1.	**a** $=\frac{1}{3^2}=\frac{1}{9}$	**b** $=1$	**c** $=\frac{1}{(-2)^6}=\frac{1}{64}$	
	d $=6^2=36$	**e** $=(\frac{3}{2})^{-3}=(\frac{2}{3})^3=\frac{8}{27}$	**f** $=\sqrt{9}=3$	
2.	**a** $=\sqrt{4}\times\sqrt[3]{27}$ $=2\times3=6$	**b** $=\sqrt[4]{16}+\sqrt{25}$ $=2+5=7$	**c** $=\frac{1}{\sqrt[3]{8}}\div\sqrt{36}$ $=\frac{1}{2}\div6=\frac{1}{12}$	**d** $=\sqrt[3]{-64}\times(\sqrt{9})^3$ $=-4\times27=-108$
	e $=3^2-\sqrt[3]{-8}$ $=9-(-2)=11$	**f** $=\sqrt{\frac{1}{25}}\times4^2$ $=\frac{1}{5}\times16=\frac{16}{5}$ or $3\frac{1}{5}$	**g** $=(\sqrt[4]{81})^3-\sqrt{49}$ $=27-7=20$	**h** $=\sqrt[3]{27}\times(\sqrt{\frac{9}{4}})^3$ $=3\times\frac{27}{8}=\frac{81}{8}$ or $10\frac{1}{8}$
3.	**a** $x=6^2=36$	**b** $x=5^3=125$	**c** $x^{\frac{1}{2}}=\frac{1}{2}$ $x=(\frac{1}{2})^2=\frac{1}{4}$	**d** $x^{\frac{1}{4}}=3$ $x=3^4=81$

4.	**a** $= x^{\frac{1}{2}}$ **b** $= x^{-\frac{1}{3}}$ **c** $= x^2 \times x^{\frac{1}{2}} = x^{\frac{5}{2}}$ **d** $= \frac{x^{\frac{1}{4}}}{x} = x^{-\frac{3}{4}}$ **e** $= (x^3)^{\frac{1}{2}} = x^{\frac{3}{2}}$ **f** $= x^{\frac{1}{2}} \times x^{\frac{1}{3}} = x^{\frac{5}{6}}$ **g** $= (x^{\frac{1}{2}})^5 = x^{\frac{5}{2}}$ **h** $= x^{\frac{2}{3}} \times x^{\frac{3}{2}} = x^{\frac{13}{6}}$
5.	**a** $4x^{-\frac{1}{2}}$ **b** $\frac{1}{2}x^{-1}$ **c** $\frac{3}{4}x^{-3}$ **d** $\frac{1}{9}x^{-2}$ **e** $\frac{2}{5}x^{-\frac{1}{3}}$ **f** $\frac{1}{3}x^{-\frac{3}{2}}$
6.	**a** $= (2^3)^2 = 2^6$ **b** $= (2^{-2})^{-2} = 2^4$ **c** $= (2^{-1})^{\frac{1}{3}} = 2^{-\frac{1}{3}}$ **d** $= (2^4)^{-\frac{1}{6}} = 2^{-\frac{2}{3}}$ **e** $= (2^3)^{\frac{2}{5}} = 2^{\frac{6}{5}}$ **f** $= (2^{-5})^{-3} = 2^{15}$

Answers – Advanced Skills

1.	**a** $= (3^2)^x = 3^{2x}$ **b** $= (3^4)^{x+1} = 3^{4x+4}$ **c** $= (3^3)^{\frac{x}{4}} = 3^{\frac{3}{4}x}$ **d** $= (3^{-1})^x = 3^{-x}$ **e** $= (3^2)^{2x-1} = 3^{4x-2}$ **f** $= (3^{-3})^{x+2} = 3^{-3x-6}$
2.	**a** $= 2 \times 2^x = 2y$ **b** $= 2^{-2} \times 2^x = \frac{1}{4}y$ **c** $= (2^x)^2 = y^2$ **d** $= (2^3)^x = 2^{3x} = (2^x)^3 = y^3$ **e** $= 2^3 \times 2^{4x} = 8y^4$ **f** $= (2^{-1})^{x-3} = 2^3 \times 2^{-x} = \frac{8}{y}$
3.	**a** $2^x = 2^6$ $x = 6$ **b** $5^{x-1} = 5^3$ $x - 1 = 3$ $x = 4$ **c** $3^{x+4} = 27 = 3^3$ $x + 4 = 3$ $x = -1$ **d** $(2^3)^x = 2^{3x} = 2$ $3x = 1$ $x = \frac{1}{3}$ **e** $3^{2x-1} = 3^2$ $2x - 1 = 2$ $x = \frac{3}{2}$ **f** $16 = 4^2 = 4^{3x-2}$ $2 = 3x - 2$ $x = \frac{4}{3}$ **g** $(3^2)^{x-2} = 3^{2x-4} = 3^3$ $2x - 4 = 3$ $x = \frac{7}{2}$ **h** $(2^3)^{2x+1} = 2^{6x+3} = 2^4$ $6x + 3 = 4$ $x = \frac{1}{6}$
4.	**a** $2^{x+3} = (2^2)^x = 2^{2x}$ $x + 3 = 2x$ $x = 3$ **b** $5^{3x} = (5^2)^{x+1} = 5^{2x+2}$ $3x = 2x + 2$ $x = 2$ **c** $(3^2)^{2x} = 3^{4x} = 3^{x-3}$ $4x = x - 3$ $x = -1$ **d** $(4^2)^x = 4^{2x} = 4^{1-x}$ $2x = 1 - x$ $x = \frac{1}{3}$ **e** $(2^2)^{x+2} = (2^3)^x$ $2^{2x+4} = 2^{3x}$ $2x + 4 = 3x$ $x = 4$ **f** $(3^3)^{2x} = (3^2)^{3-x}$ $3^{6x} = 3^{6-2x}$ $6x = 6 - 2x$ $x = \frac{3}{4}$ **g** $6^{3x-1} = (6^2)^{x+2}$ $6^{3x-1} = 6^{2x+4}$ $3x - 1 = 2x + 4$ $x = 5$ **h** $(2^3)^x = (2^4)^{2x-1}$ $2^{3x} = 2^{8x-4}$ $3x = 8x - 4$ $x = \frac{4}{5}$
5.	$25^x = (5^2)^x = 5^{4x+1}$ $5^{2x} = 5^{4x+1}$ $2x = 4x + 1$ $x = -\frac{1}{2}$
6.	**a** **i** $xy = 2^{t-1} \times 2^{3t} = 2^{4t-1}$ **ii** $2y^2 = 2 \times (2^{3t})^2 = 2 \times 2^{6t} = 2^{6t+1}$ **b** $2^{6t+1} - 2^{4t-1} = 0$ $2^{6t+1} = 2^{4t-1}$ $6t + 1 = 4t - 1$ $t = -1$

Indices Exam Questions Solutions

1. Jan 05 Q5

(i) $\left(\frac{1}{3}\right)^{-2}$

$= \left(\frac{3}{1}\right)^2$

$= 9$

(ii) $16^{3/4}$

$= (16^{1/4})^3$

$= 2^3$

$= 8$

2. Jan 05 Q6

(i) $a^0 = 1$

(ii) $a^6 \div a^{-2} = a^8$

(iii) $(9a^6b^2)^{-1/2} = \frac{1}{3}a^{-3}b^{-1}$

or $\frac{1}{3a^3b}$

3. June 06 Q9

(i) $\frac{16^{1/2}}{81^{3/4}}$

$= \frac{4}{(81^{1/4})^3}$

$= \frac{4}{3^3}$

$= \frac{4}{27}$

(ii) $\frac{12(a^3b^2c)^4}{4a^2c^6}$

$= \frac{12a^{12}b^8c^4}{4a^2c^6}$

$= 3a^{10}b^8c^{-2}$

or $\frac{3a^{10}b^8}{c^2}$

4. Jan 07 Q6

(i) $25^{3/2} = (25^{1/2})^3$

$= 5^3$

$= 125$

(ii) $\left(\frac{7}{3}\right)^{-2} = \left(\frac{3}{7}\right)^2$

$= \frac{9}{49}$

5. June 07 Q5

(i) $a^3 = 64x^{12}y^3$

$a = (64x^{12}y^3)^{1/3}$

$a = 4x^4y$

(ii) $\left(\frac{1}{2}\right)^{-5}$

$= \left(\frac{2}{1}\right)^5$

$= 32$

Topic: B2 Surds Basic Skills
https://youtu.be/jHelde32YtI

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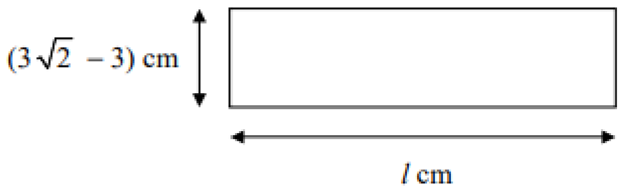
Mathematics Department

1.	Evaluate **a** $\sqrt{49}$ **b** $\sqrt{121}$ **c** $\sqrt{\frac{1}{9}}$ **d** $\sqrt{\frac{4}{25}}$ **e** $\sqrt{0.01}$ **f** $\sqrt{0.09}$
2.	Simplify **a** $\sqrt{7} \times \sqrt{7}$ **b** $4\sqrt{5} \times \sqrt{5}$ **c** $(3\sqrt{3})^2$ **d** $(\sqrt{6})^4$
3.	Simplify **a** $\sqrt{12}$ **b** $\sqrt{28}$ **c** $\sqrt{80}$ **d** $\sqrt{27}$ **e** $\sqrt{24}$ **f** $\sqrt{128}$
4.	Simplify **a** $\sqrt{18} + \sqrt{50}$ **b** $\sqrt{48} - \sqrt{27}$ **c** $2\sqrt{8} + \sqrt{72}$
5.	Express each of the following as simply as possible with a rational denominator. **a** $\frac{1}{\sqrt{5}}$ **b** $\frac{2}{\sqrt{3}}$ **c** $\frac{1}{\sqrt{8}}$ **d** $\frac{14}{\sqrt{7}}$ **e** $\frac{3\sqrt{2}}{\sqrt{3}}$ **f** $\frac{\sqrt{5}}{\sqrt{15}}$
6.	Express each of the following as simply as possible with a rational denominator. **a** $\frac{1}{\sqrt{2}+1}$ **b** $\frac{4}{\sqrt{3}-1}$ **c** $\frac{1}{\sqrt{6}-2}$ **d** $\frac{3}{2+\sqrt{3}}$

Advanced Skills

1.	Simplify **a** $(\sqrt{5} + 1)(2\sqrt{5} + 3)$ **b** $(1 - \sqrt{2})(4\sqrt{2} - 3)$ **c** $(2\sqrt{7} + 3)^2$
2.	Simplify **a** $\sqrt{8} + \frac{6}{\sqrt{2}}$ **b** $\sqrt{48} - \frac{10}{\sqrt{3}}$ **c** $\frac{6-\sqrt{8}}{\sqrt{2}}$ **d** $\frac{\sqrt{45}-5}{\sqrt{20}}$ **e** $\frac{1}{\sqrt{18}} + \frac{1}{\sqrt{32}}$ **f** $\frac{2}{\sqrt{3}} - \frac{\sqrt{6}}{\sqrt{72}}$
3.	Solve the equation $3x = \sqrt{5}(x + 2)$, giving your answer in the form $a + b\sqrt{5}$, where a and b are rational.
4.	Express each of the following as simply as possible with a rational denominator. **a** $\frac{\sqrt{2}}{\sqrt{2}+\sqrt{6}}$ **b** $\frac{1+\sqrt{3}}{2+\sqrt{3}}$ **c** $\frac{1+\sqrt{10}}{\sqrt{10}-3}$ **d** $\frac{3-\sqrt{2}}{4+3\sqrt{2}}$
5.	 The diagram shows a rectangle measuring $(3\sqrt{2} - 3)$ cm by l cm. Given that the area of the rectangle is 6 cm^2, find the exact value of l in its simplest form.

Exam Questions *(AQA Questions)*

1.	**Jan 05 Q5** (a) Simplify $(\sqrt{12}+2)(\sqrt{12}-2)$. *(2 marks)* (b) Express $\sqrt{12}$ in the form $m\sqrt{3}$, where m is an integer. *(1 mark)* (c) Express $\dfrac{\sqrt{12}+2}{\sqrt{12}-2}$ in the form $a+b\sqrt{3}$, where a and b are integers. *(4 marks)*
2.	**June 05 Q5** Express each of the following in the form $m+n\sqrt{3}$, where m and n are integers: (a) $(\sqrt{3}+1)^2$; *(2 marks)* (b) $\dfrac{\sqrt{3}+1}{\sqrt{3}-1}$. *(3 marks)*
3.	**Jan 06 Q1** (a) Simplify $(\sqrt{5}+2)(\sqrt{5}-2)$. *(2 marks)* (b) Express $\sqrt{8}+\sqrt{18}$ in the form $n\sqrt{2}$, where n is an integer. *(2 marks)*
4.	**June 06 Q4** (a) Express $(4\sqrt{5}-1)(\sqrt{5}+3)$ in the form $p+q\sqrt{5}$, where p and q are integers. *(3 marks)* (b) Show that $\dfrac{\sqrt{75}-\sqrt{27}}{\sqrt{3}}$ is an integer and find its value. *(3 marks)*
5.	**Jan 07 Q3** (a) Express $\dfrac{\sqrt{5}+3}{\sqrt{5}-2}$ in the form $p\sqrt{5}+q$, where p and q are integers. *(4 marks)* (b) (i) Express $\sqrt{45}$ in the form $n\sqrt{5}$, where n is an integer. *(1 mark)* (ii) Solve the equation $$x\sqrt{20}=7\sqrt{5}-\sqrt{45}$$ giving your answer in its simplest form. *(3 marks)*
6.	**June 07 Q7** (a) Express $\dfrac{\sqrt{63}}{3}+\dfrac{14}{\sqrt{7}}$ in the form $n\sqrt{7}$, where n is an integer. *(3 marks)* (b) Express $\dfrac{\sqrt{7}+1}{\sqrt{7}-2}$ in the form $p\sqrt{7}+q$, where p and q are integers. *(4 marks)*

Answers – Basic Skills

1.	**a** $=7$	**b** $=11$	**c** $=\frac{1}{3}$
	d $=\frac{2}{5}$	**e** $=0.1$	**f** $=0.3$
2.	**a** $=7$	**b** $=20$	**c** $=27$	**d** $=36$

3.	**a** $=\sqrt{4}\times\sqrt{3}=2\sqrt{3}$ **b** $=\sqrt{4}\times\sqrt{7}=2\sqrt{7}$ **c** $=\sqrt{16}\times\sqrt{5}=4\sqrt{5}$ **d** $=\sqrt{9}\times\sqrt{3}=3\sqrt{3}$ **e** $=\sqrt{4}\times\sqrt{6}=2\sqrt{6}$ **f** $=\sqrt{64}\times\sqrt{2}=8\sqrt{2}$
4.	**a** $=3\sqrt{2}+5\sqrt{2}=8\sqrt{2}$ **b** $=4\sqrt{3}-3\sqrt{3}=\sqrt{3}$ **c** $=4\sqrt{2}+6\sqrt{2}=10\sqrt{2}$
5.	**a** $=\frac{1}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}=\frac{1}{5}\sqrt{5}$ **b** $=\frac{2}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{2}{3}\sqrt{3}$ **c** $=\frac{1}{2\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}=\frac{1}{4}\sqrt{2}$ **d** $=\frac{14}{\sqrt{7}}\times\frac{\sqrt{7}}{\sqrt{7}}=2\sqrt{7}$ **e** $=\frac{3\sqrt{2}}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\sqrt{6}$ **f** $=\frac{\sqrt{5}}{\sqrt{3}\sqrt{5}}=\frac{1}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{1}{3}\sqrt{3}$
6.	**a** $=\frac{1}{\sqrt{2}+1}\times\frac{\sqrt{2}-1}{\sqrt{2}-1}=\frac{\sqrt{2}-1}{2-1}=\sqrt{2}-1$ **b** $=\frac{4}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}=\frac{4(\sqrt{3}+1)}{3-1}=2(\sqrt{3}+1)$ **c** $=\frac{1}{\sqrt{6}-2}\times\frac{\sqrt{6}+2}{\sqrt{6}+2}=\frac{\sqrt{6}+2}{6-4}=\frac{1}{2}(\sqrt{6}+2)$ or $\frac{1}{2}\sqrt{6}+1$ **d** $=\frac{3}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}=\frac{3(2-\sqrt{3})}{4-3}=3(2-\sqrt{3})$

Answers – Advanced Skills

1.	**a** $=10+3\sqrt{5}+2\sqrt{5}+3$ $=13+5\sqrt{5}$ **b** $=4\sqrt{2}-3-8+3\sqrt{2}$ $=7\sqrt{2}-11$ **c** $=28+12\sqrt{7}+9$ $=37+12\sqrt{7}$
2.	**a** $=2\sqrt{2}+\frac{6}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$ $=2\sqrt{2}+3\sqrt{2}$ $=5\sqrt{2}$ **b** $=4\sqrt{3}-\frac{10}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$ $=4\sqrt{3}-\frac{10}{3}\sqrt{3}$ $=\frac{2}{3}\sqrt{3}$ **c** $=\frac{6-2\sqrt{2}}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$ $=\frac{6\sqrt{2}-4}{2}$ $=3\sqrt{2}-2$ **d** $=\frac{3\sqrt{5}-5}{2\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}$ $=\frac{15-5\sqrt{5}}{10}$ $=\frac{1}{2}(3-\sqrt{5})$ **e** $=\frac{1}{3\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}+\frac{1}{4\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$ $=\frac{1}{6}\sqrt{2}+\frac{1}{8}\sqrt{2}$ $=\frac{7}{24}\sqrt{2}$ **f** $=\frac{2}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}-\frac{\sqrt{2}\sqrt{3}}{6\sqrt{2}}$ $=\frac{2}{3}\sqrt{3}-\frac{1}{6}\sqrt{3}$ $=\frac{1}{2}\sqrt{3}$
3.	$3x=\sqrt{5}x+2\sqrt{5}$ $x(3-\sqrt{5})=2\sqrt{5}$ $x=\frac{2\sqrt{5}}{3-\sqrt{5}}=\frac{2\sqrt{5}}{3-\sqrt{5}}\times\frac{3+\sqrt{5}}{3+\sqrt{5}}=\frac{2\sqrt{5}(3+\sqrt{5})}{9-5}$ $x=\frac{6\sqrt{5}+10}{4}=\frac{5}{2}+\frac{3}{2}\sqrt{5}$
4.	**a** $=\frac{\sqrt{2}}{\sqrt{2}+\sqrt{6}}\times\frac{\sqrt{2}-\sqrt{6}}{\sqrt{2}-\sqrt{6}}=\frac{\sqrt{2}(\sqrt{2}-\sqrt{6})}{2-6}=-\frac{1}{4}(2-2\sqrt{3})=\frac{1}{2}(\sqrt{3}-1)$ **b** $=\frac{1+\sqrt{3}}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}=\frac{(1+\sqrt{3})(2-\sqrt{3})}{4-3}=2-\sqrt{3}+2\sqrt{3}-3=\sqrt{3}-1$ **c** $=\frac{1+\sqrt{10}}{\sqrt{10}-3}\times\frac{\sqrt{10}+3}{\sqrt{10}+3}=\frac{(1+\sqrt{10})(\sqrt{10}+3)}{10-9}=\sqrt{10}+3+10+3\sqrt{10}=13+4\sqrt{10}$ **d** $=\frac{3-\sqrt{2}}{4+3\sqrt{2}}\times\frac{4-3\sqrt{2}}{4-3\sqrt{2}}=\frac{(3-\sqrt{2})(4-3\sqrt{2})}{16-18}=\frac{12-9\sqrt{2}-4\sqrt{2}+6}{-2}=\frac{1}{2}(13\sqrt{2}-18)$ or $\frac{13}{2}\sqrt{2}-9$
5.	$l=\frac{6}{3\sqrt{2}-3}=\frac{6}{3\sqrt{2}-3}\times\frac{3\sqrt{2}+3}{3\sqrt{2}+3}=\frac{6(3\sqrt{2}+3)}{18-9}$ $l=\frac{18(\sqrt{2}+1)}{9}=2\sqrt{2}+2$

Exam Questions Solutions – Surds

1. Jan 05 Q5

(a) $(\sqrt{12}+2)(\sqrt{12}-2)$ (M1)

$= 12 - 2\sqrt{12} + 2\sqrt{12} - 4$

$= 8$ (A1)

(b) $\sqrt{12} = \sqrt{4}\sqrt{3}$

$= 2\sqrt{3}$

(B1)

(c) $\dfrac{(\sqrt{12}+2)(\sqrt{12}+2)}{(\sqrt{12}-2)(\sqrt{12}+2)}$ (M1)

$= \dfrac{12+2\sqrt{12}+2\sqrt{12}+4}{8}$ (A1)

$= \dfrac{16+4\sqrt{12}}{8}$

$= \dfrac{16+8\sqrt{3}}{8}$ (A1)

$= 2+\sqrt{3}$ (A1)

2. June 05 Q5

(a) $(\sqrt{3}+1)^2$

$= (\sqrt{3}+1)(\sqrt{3}+1)$ (M1)

$= 3+\sqrt{3}+\sqrt{3}+1$

$= 4+2\sqrt{3}$ (A1)

(b) $\dfrac{(\sqrt{3}+1)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$ (M1)

$= \dfrac{4+2\sqrt{3}}{3+\sqrt{3}-\sqrt{3}-1}$ (A1)

$= \dfrac{4+2\sqrt{3}}{2}$

$= 2+\sqrt{3}$ (A1)

3. Jan 06 Q1

(a) $(\sqrt{5}+2)(\sqrt{5}-2)$

$= 5 - 2\sqrt{5} + 2\sqrt{5} - 4$ (M1)

$= 1$ (A1)

(b) $\sqrt{8}+\sqrt{18}$

$= \sqrt{4}\sqrt{2}+\sqrt{9}\sqrt{2}$ (M1)

$= 2\sqrt{2}+3\sqrt{2}$

$= 5\sqrt{2}$ (A1)

4. June 06 Q4

(a) $(4\sqrt{5}-1)(\sqrt{5}+3)$

$= 20+12\sqrt{5}-\sqrt{5}-3$ (M1)(A1)

$= 17+11\sqrt{5}$ (A1)

(b) $\dfrac{\sqrt{75}-\sqrt{27}}{\sqrt{3}}$

$= \dfrac{5\sqrt{3}-3\sqrt{3}}{\sqrt{3}}$ (M1)

$= \dfrac{2\sqrt{3}}{\sqrt{3}}$ (M1)

$= 2$ (A1)

5. Jan 07 Q3

(a) $\frac{(\sqrt{5}+3)(\sqrt{5}+2)}{(\sqrt{5}-2)(\sqrt{5}+2)}$ (M1)

$= \frac{5+2\sqrt{5}+3\sqrt{5}+6}{5-4}$ (A1) (A1)

$= 11+5\sqrt{5}$ (A1)

(b)(i) $\sqrt{45} = \sqrt{9}\sqrt{5}$ (B1)

$= 3\sqrt{5}$

(ii) $x\sqrt{20} = 7\sqrt{5} - \sqrt{45}$

$2x\sqrt{5} = 7\sqrt{5} - 3\sqrt{5}$ (M1)

$2x\sqrt{5} = 4\sqrt{5}$ (M1)

$2x = 4$

$x = 2$ (A1)

6. June 07 Q7

(a) $\frac{\sqrt{63}}{3} + \frac{14}{\sqrt{7}}$

(M1)

$= \frac{3\sqrt{7}}{3} + \frac{14}{\sqrt{7}}.\frac{\sqrt{7}}{\sqrt{7}}$ (M1)

$= \frac{3\sqrt{7}}{3} + \frac{14\sqrt{7}}{7}$

$= \sqrt{7} + 2\sqrt{7}$

$= 3\sqrt{7}$ (A1)

(b) $\frac{(\sqrt{7}+1)(\sqrt{7}+2)}{(\sqrt{7}-2)(\sqrt{7}+2)}$ (M1)

$= \frac{7+2\sqrt{7}+\sqrt{7}+2}{7-4}$ (A1) (A1)

$= \frac{9+3\sqrt{7}}{3}$

$= 3+\sqrt{7}$ (A1)

Topic: B3 Quadratics **Basic Skills**
https://youtu.be/Pziws8ojnlk
https://youtu.be/sn_joGVj15w
https://youtu.be/kk7p6hjn7hQ
https://youtu.be/tolqbX_NXHo

newcollege Pontefract

Mathematics Department

1.	Using factorisation, solve each equation. **a** $x^2 - 4x + 3 = 0$ **b** $x^2 + 6x + 8 = 0$ **c** $x^2 + 4x - 5 = 0$ **d** $x^2 - 7x = 8$ **e** $x^2 - 25 = 0$ **f** $x(x - 1) = 42$ **g** $x^2 = 3x$ **h** $27 + 12x + x^2 = 0$ **i** $60 - 4x - x^2 = 0$ **j** $5x + 14 = x^2$ **k** $2x^2 - 3x + 1 = 0$ **l** $x(x - 1) = 6(x - 2)$
2.	**a** $x - 5 + \frac{4}{x} = 0$ **b** $x - \frac{10}{x} = 3$ **c** $2x^3 - x^2 - 3x = 0$ **d** $x^2(10 - x^2) = 9$ **e** $\frac{5}{x^2} + \frac{4}{x} - 1 = 0$ **f** $\frac{x-6}{x-4} = x$ **g** $x + 5 = \frac{3}{x+3}$ **h** $x^2 - \frac{4}{x^2} = 3$
3.	Sketch each curve showing the coordinates of any points of intersection with the coordinate axes. **a** $y = x^2 - 3x + 2$ **b** $y = x^2 + 5x + 6$ **c** $y = x^2 - 9$ **d** $y = x^2 - 2x$ **e** $y = x^2 - 10x + 25$ **f** $y = 2x^2 - 14x + 20$
4.	Use the quadratic formula to solve each equation, giving your answers as simply as possible in terms of surds where appropriate. **a** $x^2 + 4x + 1 = 0$ **b** $4 + 8t - t^2 = 0$ **c** $y^2 - 20y + 91 = 0$ **d** $r^2 + 2r - 7 = 0$ **e** $6 + 18a + a^2 = 0$ **f** $m(m - 5) = 5$ **g** $x^2 + 11x + 27 = 0$ **h** $2u^2 + 6u + 3 = 0$ **i** $5 - y - y^2 = 0$ **j** $2x^2 - 3x = 2$ **k** $3p^2 + 7p + 1 = 0$ **l** $t^2 - 14t = 14$
5.	Express in the form $(x + a)^2 + b$ **a** $x^2 + 2x + 4$ **b** $x^2 - 2x + 4$ **c** $x^2 - 4x + 1$ **d** $x^2 + 6x$ **e** $x^2 + 4x + 8$ **f** $x^2 - 8x - 5$ **g** $x^2 + 12x + 30$ **h** $x^2 - 10x + 25$ **i** $x^2 + 6x - 9$ **j** $18 - 4x + x^2$ **k** $x^2 + 3x + 3$ **l** $x^2 + x - 1$
6.	Express in the form $a(x + b)^2 + c$ **a** $2x^2 + 4x + 3$ **b** $2x^2 - 8x - 7$ **c** $3 - 6x + 3x^2$ **d** $4x^2 + 24x + 11$ **e** $-x^2 - 2x - 5$ **f** $1 + 10x - x^2$ **g** $2x^2 + 2x - 1$ **h** $3x^2 - 9x + 5$ **i** $3x^2 - 24x + 48$ **j** $3x^2 - 15x$ **k** $70 + 40x + 5x^2$ **l** $2x^2 + 5x + 2$ **m** $4x^2 + 6x - 7$ **n** $-2x^2 + 4x - 1$ **o** $4 - 2x - 3x^2$ **p** $\frac{1}{3}x^2 + \frac{1}{2}x - \frac{1}{4}$
7.	Solve each equation by completing the square, giving your answers as simply as possible in terms of surds where appropriate. **a** $y^2 - 4y + 2 = 0$ **b** $p^2 + 2p - 2 = 0$ **c** $x^2 - 6x + 4 = 0$ **d** $7 + 10r + r^2 = 0$ **e** $x^2 - 2x = 11$ **f** $a^2 - 12a - 18 = 0$ **g** $m^2 - 3m + 1 = 0$ **h** $9 - 7t + t^2 = 0$ **i** $u^2 + 7u = 44$ **j** $2y^2 - 4y + 1 = 0$ **k** $3p^2 + 18p = -23$ **l** $2x^2 + 12x = 9$
8.	Sketch each curve showing the exact coordinates of its turning point and the point where it crosses the y-axis. **a** $y = x^2 - 4x + 3$ **b** $y = x^2 + 2x - 24$ **c** $y = x^2 - 2x + 5$ **d** $y = 30 + 8x + x^2$ **e** $y = x^2 + 2x + 1$ **f** $y = 8 + 2x - x^2$ **g** $y = -x^2 + 8x - 7$ **h** $y = -x^2 - 4x - 7$ **i** $y = x^2 - 5x + 4$ **j** $y = x^2 + 3x + 3$ **k** $y = 3 + 8x + 4x^2$ **l** $y = -2x^2 + 8x - 15$ *…include points of intesection with the x-axis*

9.	By letting $y = 2^x$, or otherwise, solve the equation $2^{2x} - 10(2^x) + 16 = 0.$

Advanced Skills

1.	Solve the equations **a** $x - \frac{5}{x} = 4$ **b** $\frac{9}{5 - x} - 1 = 2x$
2.	Find in the form $k\sqrt{2}$ the solutions of the equation $2x^2 + 5\sqrt{2}\,x - 6 = 0.$
3.	**a** Express $x^2 - 4\sqrt{2}x + 5$ in the form $a(x + b)^2 + c$. **b** Write down an equation of the line of symmetry of the curve $y = x^2 + 4\sqrt{2}x + 5$.
4.	$f(x) \equiv x^2 + 2kx - 3.$ By completing the square, find the roots of the equation $f(x) = 0$ in terms of the constant k.
5.	Labelling the coordinates of any points of intersection with the coordinate axes, sketch the curves **a** $y = (x + 1)(x - p)$ where $p > 0$, **b** $y = (x + q)^2$ where $q < 0$.
6.	$f(x) \equiv 2x^2 - 6x + 5.$ **a** Find the values of A, B and C such that $f(x) \equiv A(x + B)^2 + C.$ **b** Hence deduce the minimum value of $f(x)$.
7.	**a** Given that $t = x^{\frac{1}{3}}$ express $x^{\frac{2}{3}}$ in terms of t. **b** Hence, or otherwise, solve the equation $2x^{\frac{2}{3}} + x^{\frac{1}{3}} - 6 = 0.$
8.	**a** Given that $y = 3^x$ express 3^{2x+2} in terms of y. **b** Hence, or otherwise, solve the equation $3^{2x+2} - 10(3^x) + 1 = 0.$

Exam Questions *(AQA C1 Questions)*

1.	**Jan 2011 Q7** **(a) (i)** Express $4 - 10x - x^2$ in the form $p - (x + q)^2$. *(2 marks)* **(ii)** Hence write down the equation of the line of symmetry of the curve with equation $y = 4 - 10x - x^2$. *(1 mark)*

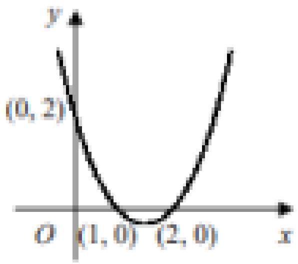
2.	**June 11 Q4** **(a)** Express $x^2 + 5x + 7$ in the form $(x+p)^2 + q$, where p and q are rational numbers. *(3 marks)* **(b)** A curve has equation $y = x^2 + 5x + 7$. **(i)** Find the coordinates of the vertex of the curve. *(2 marks)* **(ii)** State the equation of the line of symmetry of the curve. *(1 mark)* **(iii)** Sketch the curve, stating the value of the intercept on the y-axis. *(3 marks)* **(c)** Describe the geometrical transformation that maps the graph of $y = x^2$ onto the graph of $y = x^2 + 5x + 7$. *(3 marks)*
3.	**Jan 12 Q2** **(a)** Factorise $x^2 - 4x - 12$. *(1 mark)* **(b)** Sketch the graph with equation $y = x^2 - 4x - 12$, stating the values where the curve crosses the coordinate axes. *(4 marks)* **(c) (i)** Express $x^2 - 4x - 12$ in the form $(x-p)^2 - q$, where p and q are positive integers. *(2 marks)* **(ii)** Hence find the minimum value of $x^2 - 4x - 12$. *(1 mark)* **(d)** The curve with equation $y = x^2 - 4x - 12$ is translated by the vector $\begin{bmatrix} -3 \\ 2 \end{bmatrix}$. Find an equation of the new curve. You need not simplify your answer. *(2 marks)*
4.	**June 12 Q5** **(a) (i)** Express $x^2 - 3x + 5$ in the form $(x-p)^2 + q$. *(2 marks)* **(ii)** Hence write down the equation of the line of symmetry of the curve with equation $y = x^2 - 3x + 5$. *(1 mark)*
5.	**Jan 13 Q4** **(a) (i)** Express $x^2 - 6x + 11$ in the form $(x-p)^2 + q$. *(2 marks)* **(ii)** **Use the result from part (a)(i)** to show that the equation $x^2 - 6x + 11 = 0$ has no real solutions. *(2 marks)* **(b)** A curve has equation $y = x^2 - 6x + 11$. **(i)** Find the coordinates of the vertex of the curve. *(2 marks)* **(ii)** Sketch the curve, indicating the value of y where the curve crosses the y-axis. *(3 marks)* **(iii)** Describe the geometrical transformation that maps the curve with equation $y = x^2 - 6x + 11$ onto the curve with equation $y = x^2$. *(3 marks)*

6.	**June 13 Q5** **(a) (i)** Express $2x^2 + 6x + 5$ in the form $2(x+p)^2 + q$, where p and q are rational numbers. *(2 marks)* **(ii)** Hence write down the minimum value of $2x^2 + 6x + 5$. *(1 mark)*

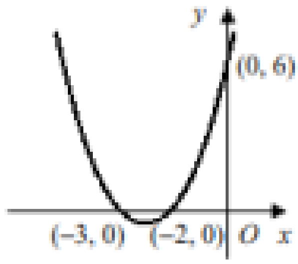
Answers – Basic Skills

1.	**a** $(x-1)(x-3)=0$ $x = 1$ or 3 **b** $(x+4)(x+2)=0$ $x=-4$ or -2 **c** $(x+5)(x-1)=0$ $x=-5$ or 1 **d** $x^2-7x-8=0$ $(x+1)(x-8)=0$ $x=-1$ or 8 **e** $(x+5)(x-5)=0$ $x=-5$ or 5 **f** $x^2-x-42=0$ $(x+6)(x-7)=0$ $x=-6$ or 7 **g** $x^2-3x=0$ $x(x-3)=0$ $x=0$ or 3 **h** $(x+9)(x+3)=0$ $x=-9$ or -3 **i** $x^2+4x-60=0$ $(x+10)(x-6)=0$ $x=-10$ or 6 **j** $x^2-5x-14=0$ $(x+2)(x-7)=0$ $x=-2$ or 7 **k** $(2x-1)(x-1)=0$ $x=\frac{1}{2}$ or 1 **l** $x^2-x=6x-12$ $x^2-7x+12=0$ $(x-3)(x-4)=0$ $x=3$ or 4
2.	**a** $x^2-5x+4=0$ $(x-1)(x-4)=0$ $x=1$ or 4 **b** $x^2-10=3x$ $x^2-3x-10=0$ $(x+2)(x-5)=0$ $x=-2$ or 5 **c** $x(2x^2-x-3)=0$ $x(2x-3)(x+1)=0$ $x=-1, 0$ or $\frac{3}{2}$ **d** $10x^2-x^4=9$ $x^4-10x^2+9=0$ $(x^2-1)(x^2-9)=0$ $x^2=1$ or 9 $x=\pm 1$ or ± 3 **e** $5+4x-x^2=0$ $x^2-4x-5=0$ $(x+1)(x-5)=0$ $x=-1$ or 5 **f** $x-6=x(x-4)$ $x-6=x^2-4x$ $x^2-5x+6=0$ $(x-2)(x-3)=0$ $x=2$ or 3 **g** $(x+5)(x+3)=3$ $x^2+8x+15=3$ $x^2+8x+12=0$ $(x+6)(x+2)=0$ $x=-6$ or -2 **h** $x^4-4=3x^2$ $x^4-3x^2-4=0$ $(x^2+1)(x^2-4)=0$ $x^2=-1$ (no sol's) or 4 $x=\pm 2$
3.	

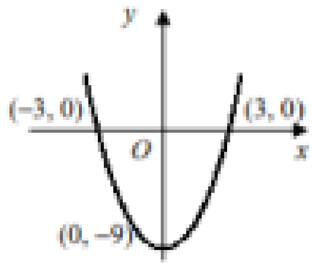
a $x^2 - 3x + 2 = 0$
$(x - 1)(x - 2) = 0$
$x = 1$ or 2



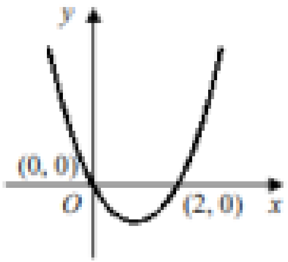
b $x^2 + 5x + 6 = 0$
$(x + 3)(x + 2) = 0$
$x = -3$ or -2



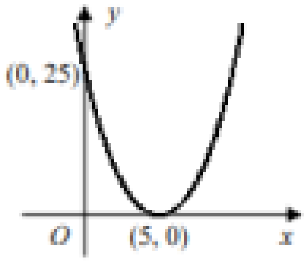
c $x^2 - 9 = 0$
$(x + 3)(x - 3) = 0$
$x = -3$ or 3



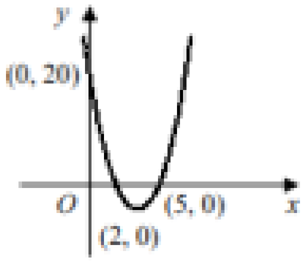
d $x^2 - 2x = 0$
$x(x - 2) = 0$
$x = 0$ or 2



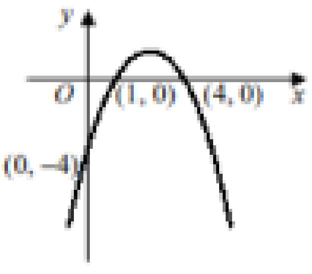
e $x^2 - 10x + 25 = 0$
$(x - 5)^2 = 0$
$x = 5$



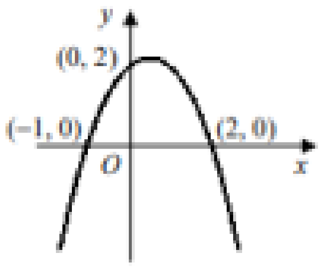
f $2x^2 - 14x + 20 = 0$
$2(x - 2)(x - 5) = 0$
$x = 2$ or 5



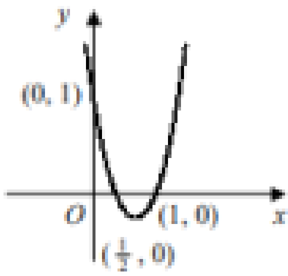
g $-x^2 + 5x - 4 = 0$
$x^2 - 5x + 4 = 0$
$(x - 1)(x - 4) = 0$
$x = 1$ or 4



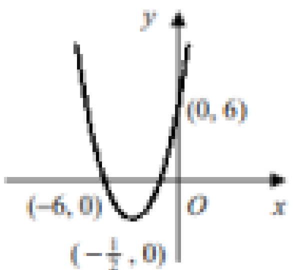
h $2 + x - x^2 = 0$
$x^2 - x - 2 = 0$
$(x + 1)(x - 2) = 0$
$x = -1$ or 2



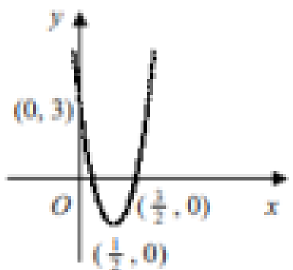
i $2x^2 - 3x + 1 = 0$
$(2x - 1)(x - 1) = 0$
$x = \frac{1}{2}$ or 1



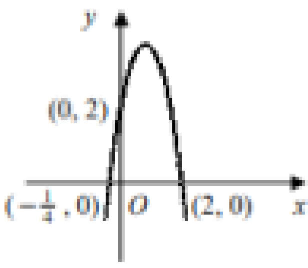
j $2x^2 + 13x + 6 = 0$
$(2x + 1)(x + 6) = 0$
$x = -6$ or $-\frac{1}{2}$



k $3 - 8x + 4x^2 = 0$
$(2x - 1)(2x - 3) = 0$
$x = \frac{1}{2}$ or $\frac{3}{2}$



l $2 + 7x - 4x^2 = 0$
$4x^2 - 7x - 2 = 0$
$(4x + 1)(x - 2) = 0$
$x = -\frac{1}{4}$ or 2



4.	**a** $x=\frac{-4\pm\sqrt{16-4}}{2}$, $x=\frac{-4\pm2\sqrt{3}}{2}$, $x=-2\pm\sqrt{3}$ **b** $t=\frac{-8\pm\sqrt{64+16}}{-2}$, $t=\frac{-8\pm4\sqrt{5}}{-2}$, $t=4\pm2\sqrt{5}$ **c** $y=\frac{20\pm\sqrt{400-364}}{2}$, $y=\frac{20\pm6}{2}$, $y=7$ or 13 **d** $r=\frac{-2\pm\sqrt{4+28}}{2}$, $r=\frac{-2\pm4\sqrt{2}}{2}$, $r=-1\pm2\sqrt{2}$ **e** $a=\frac{-18\pm\sqrt{324-24}}{2}$, $a=\frac{-18\pm10\sqrt{3}}{2}$, $a=-9\pm5\sqrt{3}$ **f** $m^2-5m-5=0$, $m=\frac{5\pm\sqrt{25+20}}{2}$, $m=\frac{1}{2}(5\pm3\sqrt{5})$ **g** $x=\frac{-11\pm\sqrt{121-108}}{2}$, $x=\frac{1}{2}(-11\pm\sqrt{13})$ **h** $u=\frac{-6\pm\sqrt{36-24}}{4}$, $u=\frac{-6\pm2\sqrt{3}}{4}$, $u=\frac{1}{2}(-3\pm\sqrt{3})$ **i** $y=\frac{1\pm\sqrt{1+20}}{-2}$, $y=-\frac{1}{2}(1\pm\sqrt{21})$ **j** $2x^2-3x-2=0$, $x=\frac{3\pm\sqrt{9+16}}{4}$, $x=\frac{3\pm5}{4}$, $x=-\frac{1}{2}$ or 2 **k** $p=\frac{-7\pm\sqrt{49-12}}{6}$, $p=\frac{1}{6}(-7\pm\sqrt{37})$ **l** $t^2-14t-14=0$, $t=\frac{14\pm\sqrt{196+56}}{2}$, $t=\frac{14\pm6\sqrt{7}}{2}$, $t=7\pm3\sqrt{7}$
5.	**a** $=(x+1)^2-1+4$, $=(x+1)^2+3$ **b** $=(x-1)^2-1+4$, $=(x-1)^2+3$ **c** $=(x-2)^2-4+1$, $=(x-2)^2-3$ **d** $=(x+3)^2-9$ **e** $=(x+2)^2-4+8$, $=(x+2)^2+4$ **f** $=(x-4)^2-16-5$, $=(x-4)^2-21$ **g** $=(x+6)^2-36+30$, $=(x+6)^2-6$ **h** $=(x-5)^2-25+25$, $=(x-5)^2$ **i** $=(x+3)^2-9-9$, $=(x+3)^2-18$ **j** $=(x-2)^2-4+18$, $=(x-2)^2+14$ **k** $=(x+\frac{3}{2})^2-\frac{9}{4}+3$, $=(x+\frac{3}{2})^2+\frac{3}{4}$ **l** $=(x+\frac{1}{2})^2-\frac{1}{4}-1$, $=(x+\frac{1}{2})^2-\frac{5}{4}$
6.	**a** $=2[x^2+2x]+3$, $=2[(x+1)^2-1]+3$, $=2(x+1)^2+1$ **b** $=2[x^2-4x]-7$, $=2[(x-2)^2-4]-7$, $=2(x-2)^2-15$ **c** $=3[x^2-2x]+3$, $=3[(x-1)^2-1]+3$, $=3(x-1)^2$ **d** $=4[x^2+6x]+11$, $=4[(x+3)^2-9]+11$, $=4(x+3)^2-25$ **e** $=-[x^2+2x]-5$, $=-[(x+1)^2-1]-5$, $=-(x+1)^2-4$ **f** $=-[x^2-10x]+1$, $=-[(x-5)^2-25]+1$, $=-(x-5)^2+26$ **g** $=2[x^2+x]-1$, $=2[(x+\frac{1}{2})^2-\frac{1}{4}]-1$, $=2(x+\frac{1}{2})^2-\frac{3}{2}$ **h** $=3[x^2-3x]+5$, $=3[(x-\frac{3}{2})^2-\frac{9}{4}]+5$, $=3(x-\frac{3}{2})^2-\frac{7}{4}$ **i** $=3[x^2-8x]+48$, $=3[(x-4)^2-16]+48$, $=3(x-4)^2$ **j** $=3[x^2-5x]$, $=3[(x-\frac{5}{2})^2-\frac{25}{4}]$, $=3(x-\frac{5}{2})^2-\frac{75}{4}$ **k** $=5[x^2+8x]+70$, $=5[(x+4)^2-16]+70$, $=5(x+4)^2-10$ **l** $=2[x^2+\frac{5}{2}x]+2$, $=2[(x+\frac{5}{4})^2-\frac{25}{16}]+2$, $=2(x+\frac{5}{4})^2-\frac{9}{8}$ **m** $=4[x^2+\frac{3}{2}x]-7$, $=4[(x+\frac{3}{4})^2-\frac{9}{16}]-7$, $=4(x+\frac{3}{4})^2-\frac{37}{4}$ **n** $=-2[x^2-2x]-1$, $=-2[(x-1)^2-1]-1$, $=-2(x-1)^2+1$ **o** $=-3[x^2+\frac{2}{3}x]+4$, $=-3[(x+\frac{1}{3})^2-\frac{1}{9}]+4$, $=-3(x+\frac{1}{3})^2+\frac{13}{3}$ **p** $=\frac{1}{3}[x^2+\frac{3}{2}x]-\frac{1}{4}$, $=\frac{1}{3}[(x+\frac{3}{4})^2-\frac{9}{16}]-\frac{1}{4}$, $=\frac{1}{3}(x+\frac{3}{4})^2-\frac{7}{16}$

7.

a $(y-2)^2-4+2=0$
$(y-2)^2=2$
$y-2=\pm\sqrt{2}$
$y=2\pm\sqrt{2}$

b $(p+1)^2-1-2=0$
$(p+1)^2=3$
$p+1=\pm\sqrt{3}$
$p=-1\pm\sqrt{3}$

c $(x-3)^2-9+4=0$
$(x-3)^2=5$
$x-3=\pm\sqrt{5}$
$x=3\pm\sqrt{5}$

d $(r+5)^2-25+7=0$
$(r+5)^2=18$
$r+5=\pm\sqrt{18}=\pm3\sqrt{2}$
$r=-5\pm3\sqrt{2}$

e $(x-1)^2-1=11$
$(x-1)^2=12$
$x-1=\pm\sqrt{12}=\pm2\sqrt{3}$
$x=1\pm2\sqrt{3}$

f $(a-6)^2-36-18=0$
$(a-6)^2=54$
$a-6=\pm\sqrt{54}=\pm3\sqrt{6}$
$a=6\pm3\sqrt{6}$

g $(m-\frac{3}{2})^2-\frac{9}{4}+1=0$
$(m-\frac{3}{2})^2=\frac{5}{4}$
$m-\frac{3}{2}=\pm\frac{\sqrt{5}}{2}$
$m=\frac{1}{2}(3\pm\sqrt{5})$

h $(t-\frac{7}{2})^2-\frac{49}{4}+9=0$
$(t-\frac{7}{2})^2=\frac{13}{4}$
$t-\frac{7}{2}=\pm\frac{\sqrt{13}}{2}$
$t=\frac{1}{2}(7\pm\sqrt{13})$

i $(u+\frac{7}{2})^2-\frac{49}{4}=44$
$(u+\frac{7}{2})^2=\frac{225}{4}$
$u+\frac{7}{2}=\pm\frac{15}{2}$
$u=-\frac{7}{2}\pm\frac{15}{2}$
$u=-11$ or 4

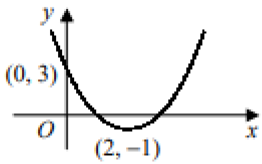
j $y^2-2y+\frac{1}{2}=0$
$(y-1)^2-1+\frac{1}{2}=0$
$(y-1)^2=\frac{1}{2}$
$y-1=\pm\frac{1}{\sqrt{2}}=\pm\frac{1}{2}\sqrt{2}$
$y=1\pm\frac{1}{2}\sqrt{2}$

k $p^2+6p=-\frac{23}{3}$
$(p+3)^2-9=-\frac{23}{3}$
$(p+3)^2=\frac{4}{3}$
$p+3=\pm\frac{2}{\sqrt{3}}=\pm\frac{2}{3}\sqrt{3}$
$p=-3\pm\frac{2}{3}\sqrt{3}$

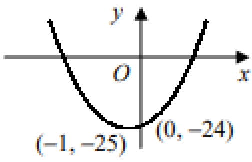
l $x^2+6x=\frac{9}{2}$
$(x+3)^2-9=\frac{9}{2}$
$(x+3)^2=\frac{27}{2}$
$x+3=\pm\sqrt{\frac{27}{2}}=\pm\frac{3}{2}\sqrt{6}$
$x=-3\pm\frac{3}{2}\sqrt{6}$

8.

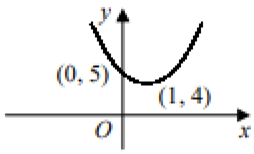
a $y=(x-2)^2-4+3$
$y=(x-2)^2-1$
minimum $(2,-1)$



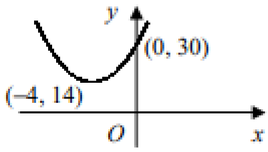
b $y=(x+1)^2-1-24$
$y=(x+1)^2-25$
minimum $(-1,-25)$



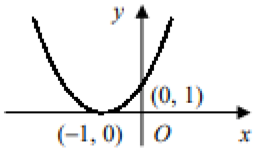
c $y=(x-1)^2-1+5$
$y=(x-1)^2+4$
minimum $(1, 4)$



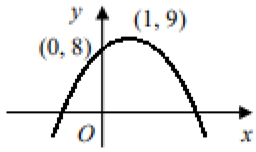
d $y=(x+4)^2-16+30$
$y=(x+4)^2+14$
minimum $(-4, 14)$



e $y=(x+1)^2-1+1$
$y=(x+1)^2$
minimum $(-1, 0)$



f $y=-[x^2-2x]+8$
$y=-[(x-1)^2-1]+8$
$y=-(x-1)^2+9$
maximum $(1, 9)$

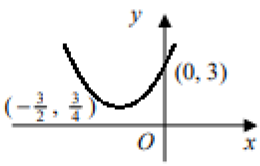


g $y=-[x^2-8x]-7$
$y=-[(x-4)^2-16]-7$
$y=-(x-4)^2+9$
maximum $(4, 9)$

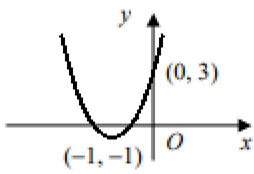
h $y=-[x^2+4x]-7$
$y=-[(x+2)^2-4]-7$
$y=-(x+2)^2-3$
maximum $(-2,-3)$

i $y=(x-\frac{5}{2})^2-\frac{25}{4}+4$
$y=(x-\frac{5}{2})^2-\frac{9}{4}$
minimum $(\frac{5}{2},-\frac{9}{4})$

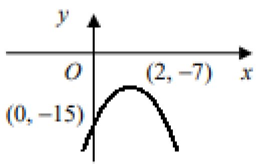
j $y=(x+\frac{3}{2})^2-\frac{9}{4}+3$
$y=(x+\frac{3}{2})^2+\frac{3}{4}$
minimum $(-\frac{3}{2},\frac{3}{4})$



k $y=4[x^2+2x]+3$
$y=4[(x+1)^2-1]+3$
$y=4(x+1)^2-1$
minimum $(-1,-1)$



l $y=-2[x^2-4x]-15$
$y=-2[(x-2)^2-4]-15$
$y=-2(x-2)^2-7$
maximum $(2,-7)$



9.

$y^2 - 10y + 16 = 0$
$(y-2)(y-8) = 0$
$y = 2^x = 2 \text{ or } 8$
$x = 1 \text{ or } 3$

Answers – Advanced Skills

1.

a $x^2 - 5 = 4x$

$x^2 - 4x - 5 = 0$

$(x+1)(x-5) = 0$

$x = -1 \text{ or } 5$

b $9 - (5 - x) = 2x(5 - x)$
$2x^2 - 9x + 4 = 0$
$(2x - 1)(x - 4) = 0$
$x = \frac{1}{2} \text{ or } 4$

2.

$$x = \frac{-5\sqrt{2} \pm \sqrt{50+48}}{4} = \frac{-5\sqrt{2} \pm \sqrt{98}}{4} = \frac{-5\sqrt{2} \pm 7\sqrt{2}}{4} = -3\sqrt{2} \text{ or } \tfrac{1}{2}\sqrt{2}$$

3.

a $= (x - 2\sqrt{2})^2 - 8 + 5$
$= (x - 2\sqrt{2})^2 - 3$

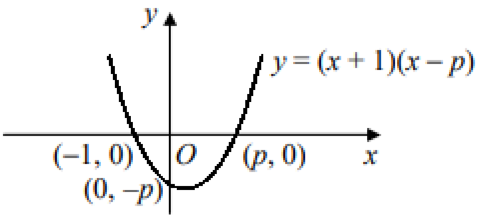
b $x = 2\sqrt{2}$

4.

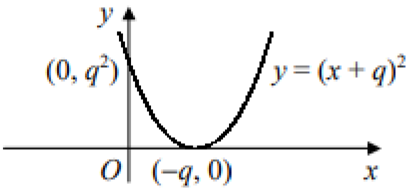
$x^2 + 2kx - 3 = 0$
$(x+k)^2 - k^2 - 3 = 0$
$(x+k)^2 = k^2 + 3$
$x + k = \pm\sqrt{k^2+3}$
$x = -k \pm \sqrt{k^2+3}$

5.

a $x = 0 \Rightarrow y = -p$
$y = 0 \Rightarrow x = -1 \text{ or } p$



b $x = 0 \Rightarrow y = q^2$
$y = 0 \Rightarrow x = -q \quad [-q > 0]$



6.

a $f(x) = 2[x^2 - 3x] + 5$
$= 2[(x - \frac{3}{2})^2 - \frac{9}{4}] + 5$
$= 2(x - \frac{3}{2})^2 + \frac{1}{2}$
$\therefore A = 2,\ B = -\frac{3}{2},\ C = \frac{1}{2}$

b minimum value of $f(x) = \frac{1}{2}$

7.

a $x^{\frac{2}{3}} = (x^{\frac{1}{3}})^2 = t^2$

b let $t = x^{\frac{1}{3}} \Rightarrow 2t^2 + t - 6 = 0$
$(2t-3)(t+2) = 0$
$t = -2 \text{ or } \frac{3}{2}$

but $x = t^3 \quad \therefore \quad x = -8 \text{ or } \frac{27}{8}$

8.	**a** $3^{2x+2} = 3^2(3^x)^2 = 9y^2$ **b** $9y^2 - 10y + 1 = 0$ $(9y-1)(y-1) = 0$ $y = 3^x = \frac{1}{9}, 1$ $\therefore\ x = -2, 0$

Quadratics Exam Questions Solutions

1. Jan 2011 Q7

(a)(i) $4 - 10x - x^2$

$\equiv -(x^2 + 10x - 4)$

$\equiv -((x+5)^2 - 25 - 4)$ (M1)

$\equiv -((x+5)^2 - 29)$

$\equiv 29 - (x+5)^2$ (A1)

(ii) line of symmetry $x = -5$ (B1FT)

2. June 11 Q4

(a) $x^2 + 5x + 7 \equiv$

$\equiv (x + 5/2)^2 - \frac{25}{4} + \frac{28}{4}$ (B1)(M1)

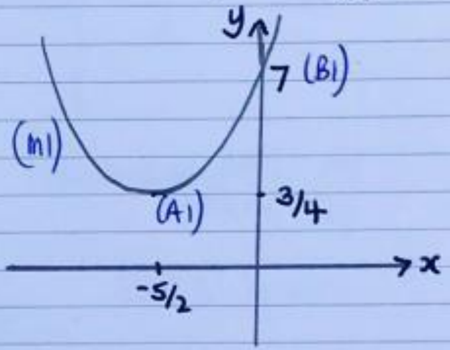
$\equiv (x + \frac{5}{2})^2 + \frac{3}{4}$ (A1)

(b) (i) when $x = -\frac{5}{2}$ $y = \frac{3}{4}$ (M1)

$\therefore$ vertex at $(-5/2, 3/4)$ (A1)

(ii) line of symmetry $x = -\frac{5}{2}$ (B1 FT)

(iii)



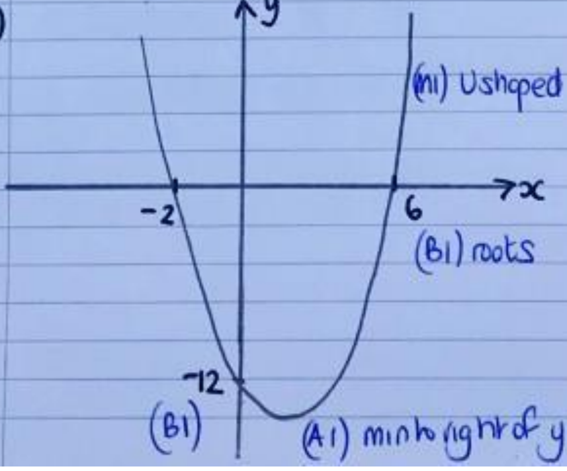
(c) Translation $\begin{pmatrix} -5/2 \\ 3/4 \end{pmatrix}$ (M1)(A1)
(E1)

3. Jan 12 Q2

(a) $x^2 - 4x - 12$

$\equiv (x-6)(x+2)$ (B1)

(b)



(c)(i) $x^2 - 4x - 12$

$\equiv (x-2)^2 - 4 - 12$ (M1)

$\equiv (x-2)^2 - 16$ (A1)

(ii) min value is -16 (B1FT)

(d) $y = (x+3)^2 - 4(x+3) - 12 + 2$ (M1)

$y = (x+3)^2 - 4(x+3) - 10$

or any equivalent form (A1)

ie $y = (x+1)^2 - 14$

4. June 12 Q5

(a)(i) $x^2 - 3x + 5$

$\equiv (x - 3/2)^2 - \frac{9}{4} + \frac{20}{4}$ (m1)

$\equiv (x - 3/2)^2 + \frac{11}{4}$ (A1)

(ii) line of symmetry is $x = 3/2$ (B1FT)

5. Jan 13 Q4

(a)(i) $x^2 - 6x + 11$

$\equiv (x-3)^2 - 9 + 11$ (m1)

$\equiv (x-3)^2 + 2$ (A1)

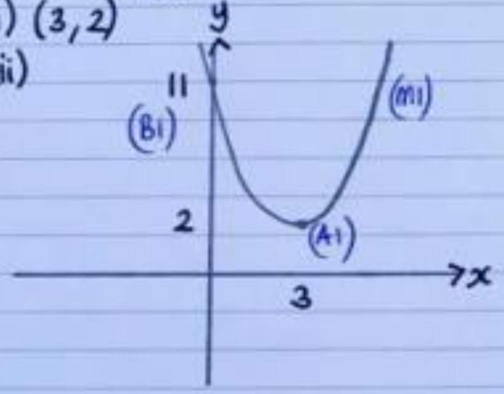
(ii) $(x-3)^2 + 2 = 0$

$(x-3)^2 = -2$ (m1)

can't take square root of negative number and get real solutions ∴ no real solutions (A1)

(b)(i) $(3, 2)$ (m1)(A1)

(b)(ii)



(iii) Translation $\begin{pmatrix} -3 \\ -2 \end{pmatrix}$ (m1) (A1) (E1)

Gone backwards !

6. June 13 Q5

(a)(i) $2x^2 + 6x + 5$

$\equiv 2[x^2 + 3x] + 5$ (m1)

$\equiv 2[(x+3/2)^2 - 9/4] + 5$

$\equiv 2(x+3/2)^2 - \frac{9}{2} + \frac{10}{2}$

$\equiv 2(x+3/2)^2 + \frac{1}{2}$ (A1)

or

$2x^2 + 6x + 5$

$\equiv 2[x^2 + 3x + 5/2]$

$\equiv 2[(x+3/2)^2 - 9/4 + 10/4]$

$\equiv 2[(x+3/2)^2 + 1/4]$

$\equiv 2(x+3/2)^2 + 1/2$

(ii) Min value is $y = 1/2$ (B1FT)

Topic: B4 Simultaneous Equations

Basic Skills

https://youtu.be/4SRtwS5unwE

Mathematics Department

1.	Solve each pair of simultaneous equations. **a** $y = 3x$, $y = 2x + 1$ **b** $y = x - 6$, $y = \frac{1}{2}x - 4$ **c** $y = 2x + 6$, $y = 3 - 4x$ **d** $x + y - 3 = 0$, $x + 2y + 1 = 0$ **e** $x + 2y + 11 = 0$, $2x - 3y + 1 = 0$ **f** $3x + 3y + 4 = 0$, $5x - 2y - 5 = 0$
2.	Solve each pair of simultaneous equations. **a** $x^2 - y + 3 = 0$, $x - y + 5 = 0$ **b** $2x^2 - y - 8x = 0$, $x + y + 3 = 0$ **c** $x^2 + y^2 = 25$, $2x - y = 5$ **d** $x^2 + 2xy + 15 = 0$, $2x - y + 10 = 0$ **e** $x^2 - 2xy - y^2 = 7$, $x + y = 1$ **f** $3x^2 - x - y^2 = 0$, $x + y - 1 = 0$ **g** $2x^2 + xy + y^2 = 22$, $x + y = 4$ **h** $x^2 - 4y - y^2 = 0$, $x - 2y = 0$ **i** $x^2 + xy = 4$, $3x + 2y = 6$
3.	Find in each case the coordinates of the points where the line l intersects the circle C. **a** $l : y = x - 4$ $C : x^2 + y^2 = 10$ **b** $l : 3x + y = 17$ $C : x^2 + y^2 - 4x - 2y - 15 = 0$ **c** $l : y = 2x + 2$ $C : 4x^2 + 4y^2 + 4x - 8y - 15 = 0$
4.	Show that the line with equation $y = 2x + 1$ is a tangent to the circle with equation $x^2 + y^2 - 8x - 8y + 27 = 0$ and find the coordinates of the point where they touch.

Advanced Skills

1.	Solve the simultaneous equations $2x^2 - y^2 - 7 = 0$ $2x - 3y + 7 = 0$		
2.	Solve each pair of simultaneous equations. **a** $x - \frac{1}{y} - 4y = 0$, $x - 6y - 1 = 0$ **b** $xy = 6$, $x - y = 5$ **c** $\frac{3}{x} - 2y + 4 = 0$, $4x + y - 7 = 0$		
3.	Solve the simultaneous equations $3^{x-1} = 9^{2y}$ $8^{x-2} = 4^{1+y}$	**4.**	Solve the simultaneous equations $4^{2x} = 2^{y-1}$ $9^{4x} = 3^{y+1}$

Exam Questions *(AQA C1 Questions)*

1.	**Jan 011 Q7** **(b)** The curve C has equation $y = 4 - 10x - x^2$ and the line L has equation $y = k(4x - 13)$, where k is a constant. **(i)** Show that the x-coordinates of any points of intersection of the curve C with the line L satisfy the equation $x^2 + 2(2k + 5)x - (13k + 4) = 0$ *(1 mark)*
2.	**Jan 13 Q8** A curve has equation $y = 2x^2 - x - 1$ and a line has equation $y = k(2x - 3)$, where k is a constant. **(a)** Show that the x-coordinate of any point of intersection of the curve and the line satisfies the equation $2x^2 - (2k + 1)x + 3k - 1 = 0$ *(1 mark)*

Answers – Basic Skills

1.

a $3x = 2x + 1$
$x = 1$
$\therefore\ x = 1, y = 3$

b $x - 6 = \frac{1}{2}x - 4$
$x = 4$
$\therefore\ x = 4, y = -2$

c $2x + 6 = 3 - 4x$
$x = -\frac{1}{2}$
$\therefore\ x = -\frac{1}{2}, y = 5$

d subtracting
$y + 4 = 0$
$y = -4$
$\therefore\ x = 7, y = -4$

e $2x + 4y + 22 = 0$
$2x - 3y + 1 = 0$
subtracting
$7y + 21 = 0$
$y = -3$
$\therefore\ x = -5, y = -3$

f $6x + 6y + 8 = 0$
$15x - 6y - 15 = 0$
adding
$21x - 7 = 0$
$x = \frac{1}{3}$
$\therefore\ x = \frac{1}{3}, y = -\frac{5}{3}$

2.

a subtracting
$x^2 - x - 2 = 0$
$(x + 1)(x - 2) = 0$
$x = -1$ or 2
$\therefore\ x = -1, y = 4$
or $x = 2, y = 7$

b adding
$2x^2 - 7x + 3 = 0$
$(2x - 1)(x - 3) = 0$
$x = \frac{1}{2}$ or 3
$\therefore\ x = \frac{1}{2}, y = -\frac{7}{2}$
or $x = 3, y = -6$

c $y = 2x - 5$
sub
$x^2 + (2x - 5)^2 = 25$
$x^2 - 4x = 0$
$x(x - 4) = 0$
$x = 0$ or 4
$\therefore\ x = 0, y = -5$
or $x = 4, y = 3$

d $y = 2x + 10$
sub.
$x^2 + 2x(2x + 10) + 15 = 0$
$x^2 + 4x + 3 = 0$
$(x + 3)(x + 1) = 0$
$x = -3$ or -1
$\therefore\ x = -3, y = 4$
or $x = -1, y = 8$

e $y = 1 - x$
sub.
$x^2 - 2x(1 - x) - (1 - x)^2 = 7$
$x^2 = 4$
$x = \pm 2$
$\therefore\ x = -2, y = 3$
or $x = 2, y = -1$

f $y = 1 - x$
sub.
$3x^2 - x - (1 - x)^2 = 0$
$2x^2 + x - 1 = 0$
$(2x - 1)(x + 1) = 0$
$x = -1$ or $\frac{1}{2}$
$\therefore\ x = -1, y = 2$
or $x = \frac{1}{2}, y = \frac{1}{2}$

g $y = 4 - x$
sub.
$2x^2 + x(4 - x) + (4 - x)^2 = 22$
$x^2 - 2x - 3 = 0$
$(x + 1)(x - 3) = 0$
$x = -1$ or 3
$\therefore\ x = -1, y = 5$
or $x = 3, y = 1$

h $x = 2y$
sub.
$(2y)^2 - 4y - y^2 = 0$
$3y^2 - 4y = 0$
$y(3y - 4) = 0$
$y = 0$ or $\frac{4}{3}$
$\therefore\ x = 0, y = 0$
or $x = \frac{8}{3}, y = \frac{4}{3}$

i $y = 3 - \frac{3}{2}x$
sub.
$x^2 + x(3 - \frac{3}{2}x) = 4$
$x^2 - 6x + 8 = 0$
$(x - 2)(x - 4) = 0$
$x = 2$ or 4
$\therefore\ x = 2, y = 0$
or $x = 4, y = -3$

3.	**a** sub. $x^2 + (x-4)^2 = 10$ $x^2 - 4x + 3 = 0$ $(x-1)(x-3) = 0$ $x = 1, 3$ $\therefore$ $(1, -3)$ and $(3, -1)$ **b** sub. $y = 17 - 3x$ $x^2 + (17-3x)^2 - 4x - 2(17-3x) - 15 = 0$ $x^2 - 10x + 24 = 0$ $(x-4)(x-6) = 0$ $x = 4, 6$ $\therefore$ $(4, 5)$ and $(6, -1)$ **c** sub. $4x^2 + 4(2x+2)^2 + 4x - 8(2x+2) - 15 = 0$ $4x^2 + 4x - 3 = 0$ $(2x+3)(2x-1) = 0$ $x = -\frac{3}{2}, \frac{1}{2}$ $\therefore$ $(-\frac{3}{2}, -1)$ and $(\frac{1}{2}, 3)$
4.	sub. $x^2 + (2x+1)^2 - 8x - 8(2x+1) + 27 = 0$ $x^2 - 4x + 4 = 0$ $(x-2)^2 = 0$ repeated root $\therefore$ tangent touch when $x = 2$ $\therefore$ at $(2, 5)$

Answers – Advanced Skills

1.	$y = \frac{2x+7}{3}$ sub. $2x^2 - (\frac{2x+7}{3})^2 - 7 = 0$ $18x^2 - (2x+7)^2 - 63 = 0$ $x^2 - 2x - 8 = 0$ $(x+2)(x-4) = 0$ $x = -2$ or 4 $\therefore$ $x = -2, y = 1$ or $x = 4, y = 5$	**3.**	$3^{x-1} = (3^2)^{2y}$ $\therefore$ $x - 1 = 4y$ $(2^3)^{x-2} = (2^2)^{1+y}$ $\therefore$ $3x - 6 = 2 + 2y$ $6x - 16 = 4y$ $\Rightarrow$ $6x - 16 = x - 1$ $x = 3$ $\therefore$ $x = 3, y = \frac{1}{2}$	**4.**	$(2^2)^{2x} = 2^{y-1}$ $4x = y - 1$ (1) $(3^2)^{4x} = 3^{y+1}$ $8x = y + 1$ (2) (1) and (2) $\Rightarrow$ $y = 4x + 1 = 8x - 1$ $4x = 2$ $x = \frac{1}{2}, y = 3$

2.	**a** subtracting $-\frac{1}{y} + 2y + 1 = 0$ $-1 + 2y^2 + y = 0$ $2y^2 + y - 1 = 0$ $(2y-1)(y+1) = 0$ $y = -1$ or $\frac{1}{2}$ $\therefore$ $x = -5, y = -1$ or $x = 4, y = \frac{1}{2}$	**b** $y = x - 5$ sub. $x(x-5) = 6$ $x^2 - 5x - 6 = 0$ $(x+1)(x-6) = 0$ $x = -1$ or 6 $\therefore$ $x = -1, y = -6$ or $x = 6, y = 1$	**c** $y = 7 - 4x$ sub. $\frac{3}{x} - 2(7-4x) + 4 = 0$ $3 - 2x(7-4x) + 4x = 0$ $8x^2 - 10x + 3 = 0$ $(4x-3)(2x-1) = 0$ $x = \frac{1}{2}$ or $\frac{3}{4}$ $\therefore$ $x = \frac{1}{2}, y = 5$ or $x = \frac{3}{4}, y = 4$

Simultaneous Equations Exam Questions

1. Jan 11 Q7

$$y = 4 - 10x - x^2 \qquad y = k(4x - 13)$$

$$k(4x - 13) = 4 - 10x - x^2$$

$$x^2 + 10x + 4kx - 13k - 4 = 0$$

$$x^2 + 2(2k+5)x - (13k+4) = 0$$

2. Jan 13 Q8

$$y = 2x^2 - x - 1 \qquad y = k(2x - 3)$$

$$2x^2 - x - 1 = k(2x - 3)$$
$$2x^2 - x - 1 = 2kx - 3k$$

$$2x^2 - 2kx - x + 3k - 1 = 0$$

$$2x^2 - (2k+1)x + 3k - 1 = 0$$

Topic: B5 Inequalities **Basic Skills**

https://youtu.be/wDut-In_7Wg

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Mathematics Department

1. Solve each inequality.

a $2y - 3 > y + 4$ **b** $5p + 1 \leq p + 3$ **c** $x - 2 < 3x - 8$

d $a + 11 \geq 15 - a$ **e** $17 - 2u < 2 + u$ **f** $5 - b \geq 14 - 3b$

g $4x + 23 < x + 5$ **h** $12 + 3y \geq 2y - 1$ **i** $16 - 3p \leq 36 + p$

2. Find the set of values of x for which

a $x^2 - 4x + 3 < 0$ **b** $x^2 - 4 \leq 0$ **c** $15 + 8x + x^2 < 0$ **d** $x^2 + 2x \leq 8$

e $x^2 - 6x + 5 > 0$ **f** $x^2 + 4x > 12$ **g** $x^2 + 10x + 21 \geq 0$ **h** $22 + 9x - x^2 > 0$

i $63 - 2x - x^2 \leq 0$ **j** $x^2 + 11x + 30 > 0$ **k** $30 + 7x - x^2 > 0$ **l** $x^2 + 91 \geq 20x$

3. Solve each inequality.

a $2x^2 - 9x + 4 \leq 0$ **b** $2r^2 - 5r - 3 < 0$ **c** $2 - p - 3p^2 \geq 0$

d $2y^2 + 9y - 5 > 0$ **e** $4m^2 + 13m + 3 < 0$ **f** $9x - 2x^2 \leq 10$

g $a^2 + 6 < 8a - 9$ **h** $x(x + 4) \leq 7 - 2x$ **i** $y(y + 9) > 2(y - 5)$

4. Giving your answers in terms of surds, find the set of values of x for which

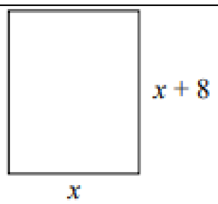
a $x^2 + 2x - 1 < 0$ **b** $x^2 - 6x + 4 > 0$ **c** $11 - 6x - x^2 > 0$ **d** $x^2 + 4x + 1 \geq 0$

5. Find the set of integers, n, for which

$$2n^2 - 5n < 12.$$

Advanced Skills

1.



The diagram shows a rectangular birthday card which is x cm wide and $(x + 8)$ cm tall.

Given that the height of the card is to be at least 50% more than its width,

a show that $x \leq 16$.

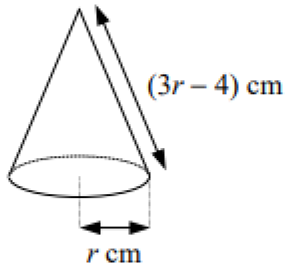
Given also that the area of the front of the card is to be at least 180 cm^2,

b find the set of possible values of x.

2. Given that $x - y = 8$,

and that $xy \leq 240$,

find the maximum value of $(x + y)$.

3.	 A party hat is designed in the shape of a right circular cone of base radius r cm and slant height $(3r-4)$ cm. Given that the height of the cone must not be more than 24 cm, find the maximum value of r.

Exam Questions *(AQA C1 Questions)*

1.	**Jan 11 Q7** **(iii)** Solve the inequality $4k^2 + 33k + 29 > 0$. *(4 marks)*
2.	**June 11 Q7** Solve each of the following inequalities: **(a)** $2(4-3x) > 5 - 4(x+2)$; *(2 marks)* **(b)** $2x^2 + 5x \geqslant 12$. *(4 marks)*
3.	**Jan 12 Q6** A rectangular garden is to have width x metres and length $(x+4)$ metres. **(a)** The perimeter of the garden needs to be greater than 30 metres. Show that $2x > 11$. *(1 mark)* **(b)** The area of the garden needs to be less than 96 square metres. Show that $x^2 + 4x - 96 < 0$. *(1 mark)* **(c)** Solve the inequality $x^2 + 4x - 96 < 0$. *(4 marks)* **(d)** Hence determine the possible values of the width of the garden. *(1 mark)*
4.	**June 12 Q7a** **(ii)** Solve the inequality $3x^2 - 10x + 8 < 0$. *(4 marks)*

Answers – Basic Skills

1.

a $y > 7$

b $4p \le 2$
$p \le \frac{1}{2}$

c $6 < 2x$
$x > 3$

d $2a \ge 4$
$a \ge 2$

e $15 < 3u$
$u > 5$

f $2b \ge 9$
$b \ge \frac{9}{2}$

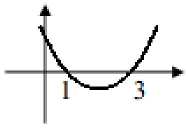
g $3x < -18$
$x < -6$

h $y \ge -13$

i $-20 \le 4p$
$p \ge -5$

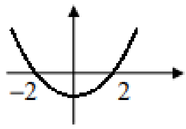
2.

a $(x-1)(x-3) < 0$



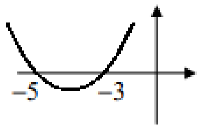
$\therefore 1 < x < 3$

b $(x+2)(x-2) \le 0$



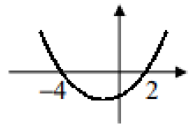
$\therefore -2 \le x \le 2$

c $(x+5)(x+3) < 0$



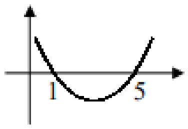
$\therefore -5 < x < -3$

d $x^2 + 2x - 8 \le 0$
$(x+4)(x-2) \le 0$



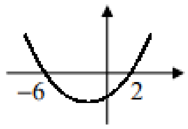
$\therefore -4 \le x \le 2$

e $(x-1)(x-5) > 0$



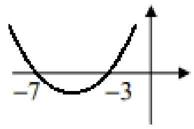
$\therefore x < 1$ or $x > 5$

f $x^2 + 4x - 12 > 0$
$(x+6)(x-2) > 0$



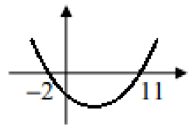
$\therefore x < -6$ or $x > 2$

g $(x+7)(x+3) \ge 0$



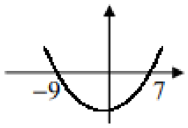
$\therefore x \le -7$ or $x \ge -3$

h $x^2 - 9x - 22 < 0$
$(x+2)(x-11) < 0$



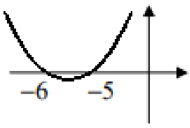
$\therefore -2 < x < 11$

i $x^2 + 2x - 63 \ge 0$
$(x+9)(x-7) \ge 0$



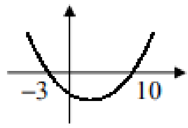
$\therefore x \le -9$ or $x \ge 7$

j $(x+6)(x+5) > 0$



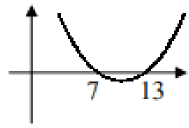
$\therefore x < -6$ or $x > -5$

k $x^2 - 7x - 30 < 0$
$(x+3)(x-10) < 0$



$\therefore -3 < x < 10$

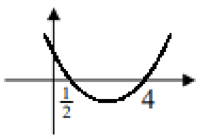
l $x^2 - 20x + 91 \ge 0$
$(x-7)(x-13) \ge 0$



$\therefore x \le 7$ or $x \ge 13$

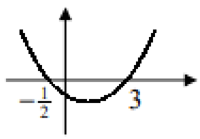
3.

a $(2x-1)(x-4) \le 0$



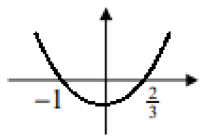
$\therefore \frac{1}{2} \le x \le 4$

b $(2r+1)(r-3) < 0$



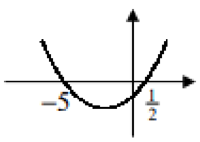
$\therefore -\frac{1}{2} < r < 3$

c $3p^2 + p - 2 \le 0$
$(3p-2)(p+1) \le 0$



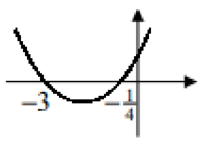
$\therefore -1 \le p \le \frac{2}{3}$

d $(2y-1)(y+5) > 0$



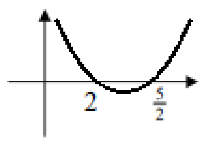
$\therefore y < -5$ or $y > \frac{1}{2}$

e $(4m+1)(m+3) < 0$



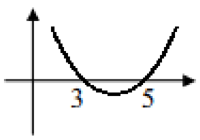
$\therefore -3 < m < -\frac{1}{4}$

f $2x^2 - 9x + 10 \ge 0$
$(2x-5)(x-2) \ge 0$



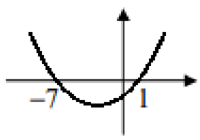
$\therefore x \le 2$ or $x \ge \frac{5}{2}$

g $a^2 - 8a + 15 < 0$
$(a-3)(a-5) < 0$



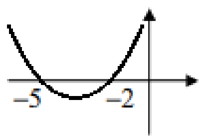
$\therefore 3 < a < 5$

h $x^2 + 4x \le 7 - 2x$
$x^2 + 6x - 7 \le 0$
$(x+7)(x-1) \le 0$



$\therefore -7 \le x \le 1$

i $y^2 + 9y > 2y - 10$
$y^2 + 7y + 10 > 0$
$(y+5)(y+2) > 0$



$\therefore y < -5$ or $y > -2$

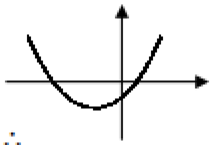
4.

a for critical values

$x = \frac{-2 \pm \sqrt{4+4}}{2}$

$x = \frac{-2 \pm 2\sqrt{2}}{2}$

$x = -1 \pm \sqrt{2}$



$\therefore$

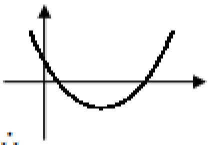
$-1-\sqrt{2} < x < -1+\sqrt{2}$

b for critical values

$x = \frac{6 \pm \sqrt{36-16}}{2}$

$x = \frac{6 \pm 2\sqrt{5}}{2}$

$x = 3 \pm \sqrt{5}$



$\therefore$

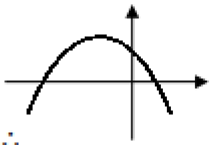
$x < 3-\sqrt{5}$ or $x > 3+\sqrt{5}$

c for critical values

$x = \frac{6 \pm \sqrt{36+44}}{-2}$

$x = \frac{6 \pm 4\sqrt{5}}{-2}$

$x = -3 \pm 2\sqrt{5}$



$\therefore$

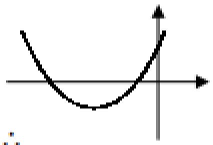
$-3-2\sqrt{5} < x < -3+2\sqrt{5}$

d for critical values

$x = \frac{-4 \pm \sqrt{16-4}}{2}$

$x = \frac{-4 \pm 2\sqrt{3}}{2}$

$x = -2 \pm \sqrt{3}$



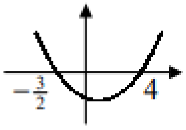
$\therefore$

$x \le -2-\sqrt{3}$ or $x \ge -2+\sqrt{3}$

5.

$2n^2 - 5n - 12 < 0$

$(2n+3)(n-4) < 0$



$-\frac{3}{2} < n < 4$

n integer $\therefore$ $n = -1, 0, 1, 2, 3$

Answers – Advanced Skills

1.

a $(x+8) \ge 1.5 \times x$

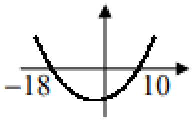
$8 \ge 0.5x$

$x \le 16$

b $x(x+8) \ge 180$

$x^2 + 8x - 180 \ge 0$

$(x+18)(x-10) \ge 0$



$x \le -18$ or $x \ge 10$

but $x > 0$ (width > 0)

and $x \le 16$ $\therefore$ $10 \le x \le 16$

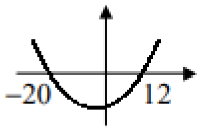
2.

$x = y + 8$

sub. $y(y+8) \le 240$

$y^2 + 8y - 240 \le 0$

$(y+20)(y-12) \le 0$



$-20 \le y \le 12$

$x + y = y + 8 + y = 2y + 8$

$\therefore$ max value of $(x+y) = 2(12) + 8 = 32$

3.

let height be h $\therefore$ $h^2 = (3r-4)^2 - r^2$

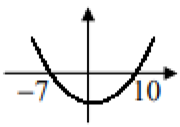
but $h \le 24$

$\therefore$ $h^2 \le 24^2$

$(3r-4)^2 - r^2 \le 576$

$r^2 - 3r - 70 \le 0$

$(r+7)(r-10) \le 0$



$-7 \le r \le 10$

$\therefore$ maximum value of $r = 10$

Inequalities Exam Questions Solutions

1. Jan 11 Q7 (ii)

$4k^2 + 33k + 29 > 0$

$(4k + 29)(k+1) > 0$ (M1)

CVs at $k = -\frac{29}{4}$ $k = -1$ (A1)

(M1)

$k < -\frac{29}{4}$ or $k > -1$ (A1)

/4

2. June 11 Q7

(a) $2(4-3x) > 5 - 4(x+2)$

$8 - 6x > 5 - 4x - 8$ (M1)

$-2x > -11$

$2x < 11$ ($\times -1$ reverse inequality sign)

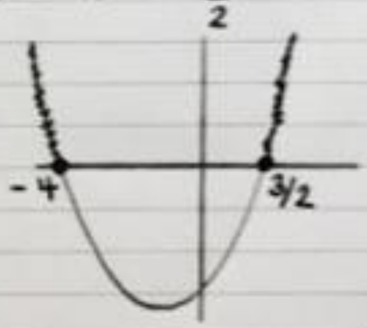
$x < \frac{11}{2}$ (A1)

(b) $2x^2 + 5x \geq 12$

$2x^2 + 5x - 12 \geq 0$

$(2x-3)(x+4) \geq 0$ (M1)

CVs at $x = \frac{3}{2}$ $x = -4$ (A1)

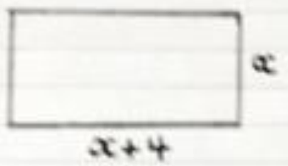


(M1)

$x \leq -4$ or $x \geq 3/2$ (A1)

/6

3 Jan 12 Q6



(a) $x + x + x + 4 + x + 4 > 30$

$4x + 8 > 30$

$4x > 22$

$2x > 11$ (B1)

$(x > \frac{11}{2})$

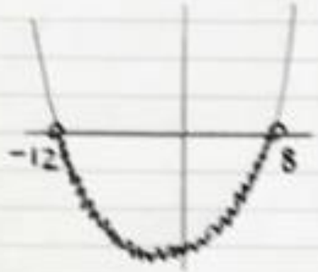
(b) $x(x+4) < 96$

$x^2 + 4x < 96$

$x^2 + 4x - 96 < 0$ (B1)

$(x+12)(x-8) < 0$ (M1)

cvs $x = -12$ $x = 8$ (A1)



(M1)

/7

$-12 < x < 8$ (A1)

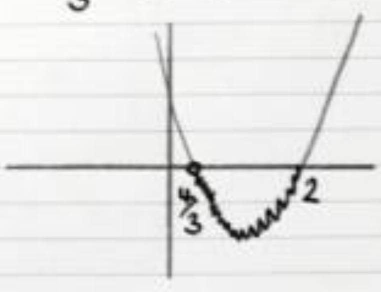
(c) $\frac{11}{2} < x < 8$ (B1)

4. June 12 Q7(a)

$3x^2 - 10x + 8 < 0$

$(3x-4)(x-2) < 0$ (M1)

cvs $x = \frac{4}{3}$ $x = 2$ (A1)



(M1)

/4

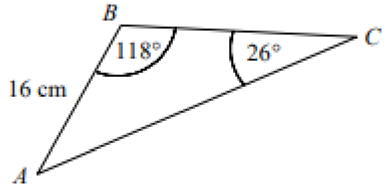
$\frac{4}{3} < x < 2$ (A1)

Topic: E1 Triangle Geometry (Trigonometry) **Basic Skills**

https://youtu.be/uVl6TAb0vBg

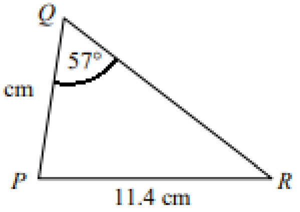
Mathematics Department

1.



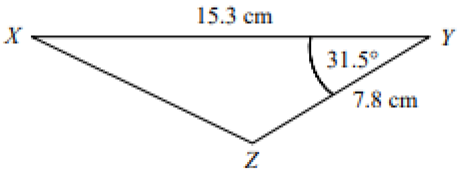
The diagram shows triangle ABC in which $AB = 16$ cm, $\angle ABC = 118°$ and $\angle ACB = 26°$.
Use the sine rule to find the length AC to 3 significant figures.

2.



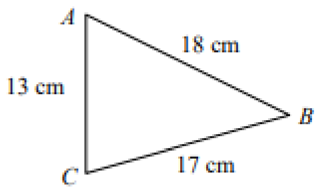
The diagram shows triangle PQR in which $PQ = 8.2$ cm, $PR = 11.4$ cm and $\angle PQR = 57°$.
Use the sine rule to find the size of $\angle PRQ$ in degrees to 1 decimal place.

3.



The diagram shows triangle XYZ in which $XY = 15.3$ cm, $YZ = 7.8$ cm and $\angle XYZ = 31.5°$.
Use the cosine rule to find the length XZ.

4.

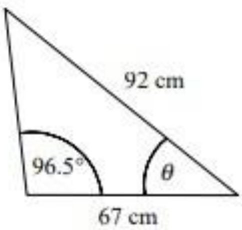


The diagram shows triangle ABC in which $AB = 18$ cm, $AC = 13$ cm and $BC = 17$ cm.
Use the cosine rule to find the size of $\angle ACB$.

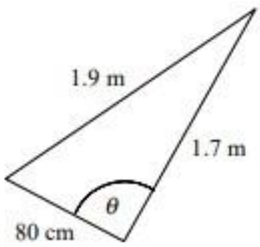
5.

Find the angle θ in each triangle.

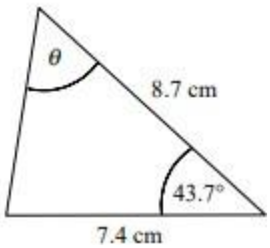
a



b



c



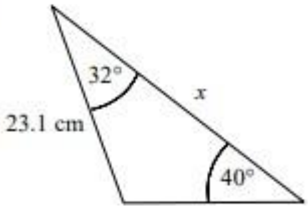
Advanced Skills

1. In triangle ABC, $AB = 16.2$ cm, $BC = 12.3$ cm and $\angle BAC = 37°$.

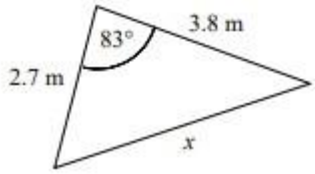
Find the two possible sizes of $\angle ACB$ and the corresponding lengths of AC.

2. Find the length x in each triangle.

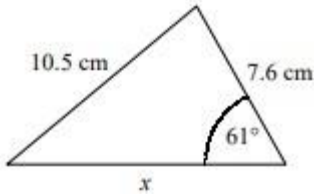
a



b

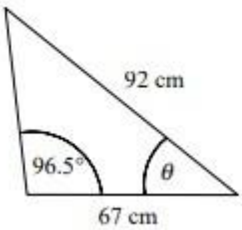


c

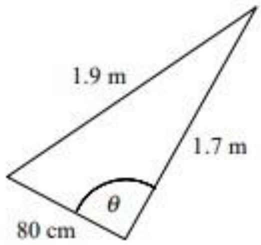


3. Find the angle θ in each triangle.

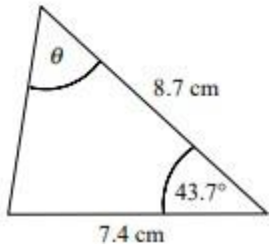
a



b



c



4. Joanne walks 4.2 miles on a bearing of 138°. She then walks 7.8 miles on a bearing of 251°.

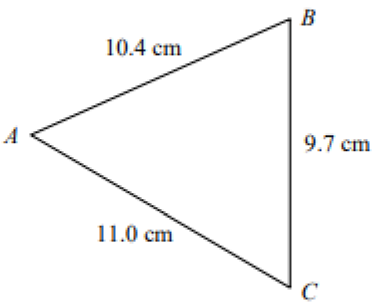
a Calculate how far Joanne is from the point where she started.

b Find, as a bearing, the direction in which Joanne would have to walk in order to return to the point where she started.

5. A ferry and a cargo ship are both approaching the same port. The ferry is 3.2 km from the port on a bearing of 076° and the cargo ship is 6.9 km from the port on a bearing of 323°.

Find the distance between the two vessels and the bearing of the cargo ship from the ferry.

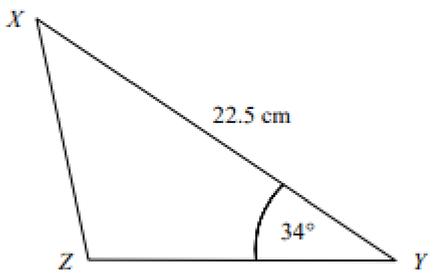
6.



The diagram shows triangle ABC in which $AB = 10.4$ cm, $AC = 11.0$ cm and $BC = 9.7$ cm.

Find the area of the triangle to 3 significant figures.

7.



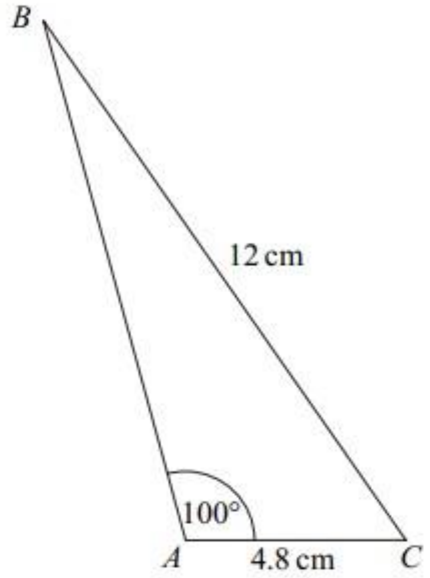
The diagram shows triangle XYZ in which $XY = 22.5$ cm and $\angle XYZ = 34°$.

Given that the area of the triangle is 100 cm^2, find the length XZ.

Exam Questions *(AQA C2 Questions)*

1. **June 2006 Q2**

The diagram shows a triangle ABC.



The lengths of AC and BC are 4.8 cm and 12 cm respectively.

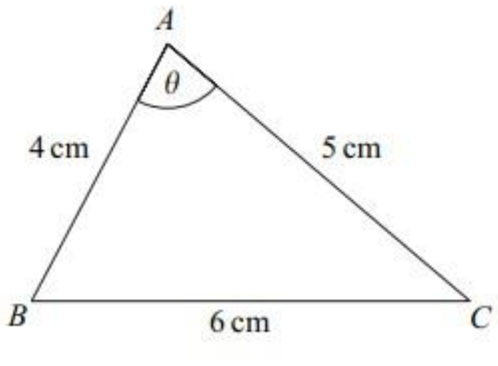
The size of the angle BAC is $100°$.

(a) Show that angle $ABC = 23.2°$, correct to the nearest $0.1°$. *(3 marks)*

(b) Calculate the area of triangle ABC, giving your answer in cm^2 to three significant figures. *(3 marks)*

2. **Jan 2007 Q4 (adapted)**

The triangle ABC, shown in the diagram, is such that $BC = 6$ cm, $AC = 5$ cm and $AB = 4$ cm. The angle BAC is θ.

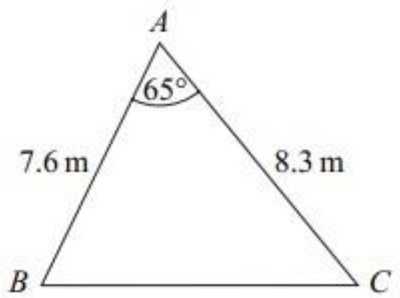


(a) Use the cosine rule to show that $\cos\theta = \frac{1}{8}$. *(3 marks)*

(c) Hence find the area of the triangle ABC. *(2 marks)*

3. **June 2008 Q4**

The diagram shows a triangle ABC.



The size of angle BAC is 65°, and the lengths of AB and AC are 7.6 m and 8.3 m respectively.

(a) Show that the length of BC is 8.56 m, correct to three significant figures. *(3 marks)*

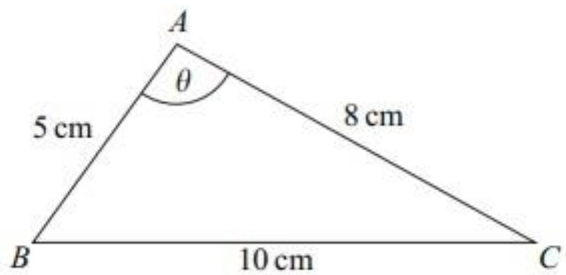
(b) Calculate the area of triangle ABC, giving your answer in m^2 to three significant figures. *(2 marks)*

(c) The perpendicular from A to BC meets BC at the point D.

Calculate the length of AD, giving your answer to the nearest 0.1 m. *(3 marks)*

4. **Jan 2011 Q3**

The triangle ABC, shown in the diagram, is such that $AB = 5$ cm, $AC = 8$ cm, $BC = 10$ cm and angle $BAC = \theta$.



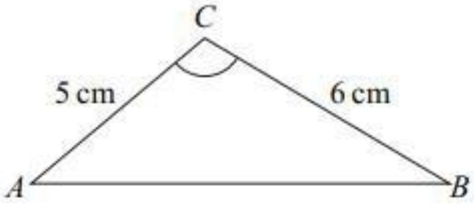
(a) Show that $\theta = 97.9°$, correct to the nearest 0.1°. *(3 marks)*

(b) (i) Calculate the area of triangle ABC, giving your answer, in cm^2, to three significant figures. *(2 marks)*

(ii) The line through A, perpendicular to BC, meets BC at the point D. Calculate the length of AD, giving your answer, in cm, to three significant figures. *(3 marks)*

5. **Jan 2013 Q3**

The diagram shows a triangle ABC.



The lengths of AC and BC are 5 cm and 6 cm respectively.

The area of triangle ABC is 12.5 cm^2, and angle ACB is **obtuse**.

(a) Find the size of angle ACB, giving your answer to the nearest 0.1°. *(3 marks)*

(b) Find the length of AB, giving your answer to two significant figures. *(3 marks)*

Answers – Basic Skills

1.

$$\frac{AC}{\sin 118} = \frac{16}{\sin 26}$$

$$AC = \frac{16 \times \sin 118}{\sin 26}$$

$$= 32.2 \text{ cm}$$

2.

$$\frac{\sin \angle PRQ}{8.2} = \frac{\sin 57}{11.4}$$

$$\sin \angle PRQ = \frac{8.2 \times \sin 57}{11.4} = 0.6033$$

$$\angle PRQ = 37.1°$$

3.

$$XZ^2 = 7.8^2 + 15.3^2 - (2 \times 7.8 \times 15.3 \times \cos 31.5°)$$

$$= 91.422$$

$$XZ = 9.56 \text{ cm (3sf)}$$

4.

$$18^2 = 13^2 + 17^2 - (2 \times 13 \times 17 \times \cos \angle ACB)$$

$$\cos \angle ACB = \frac{13^2 + 17^2 - 18^2}{2 \times 13 \times 17}$$

$$= 0.3032$$

$$\angle ACB = 72.4° \text{ (1dp)}$$

5.

a area

$$= \tfrac{1}{2} \times 2.1 \times 3.4 \times \sin 66$$

$$= 3.26 \text{ m}^2 \text{ (3sf)}$$

b area

$$= \tfrac{1}{2} \times 35 \times 68 \times \sin 116$$

$$= 1070 \text{ cm}^2 \text{ (3sf)}$$

c

$$\frac{\sin \alpha}{5.8} = \frac{\sin 72.4}{6.5}$$

$$\sin \alpha = \frac{5.8 \times \sin 72.4}{6.5} = 0.8505$$

$$\alpha = 58.270$$

$$\beta = 180 - (72.4 + \alpha) = 49.330$$

area

$$= \tfrac{1}{2} \times 5.8 \times 6.5 \times \sin 49.330$$

$$= 14.3 \text{ cm}^2 \text{ (3sf)}$$

Answers – Advanced Skills

1.

$$\frac{\sin \angle ACB}{16.2} = \frac{\sin 37}{12.3}$$

$$\sin \angle ACB = \frac{16.2 \times \sin 37}{12.3} = 0.7926$$

$$\angle ACB = 52.4 \text{ or } 180 - 52.4 = 52.4 \text{ or } 127.6$$

$$\angle ABC = 180 - (37 + \angle ACB) = 90.568 \text{ or } 15.432$$

$$\frac{AC}{\sin \angle ABC} = \frac{12.3}{\sin 37}$$

$$AC = \frac{12.3 \times \sin \angle ABC}{\sin 37} = 20.4 \text{ or } 5.4$$

$$\therefore\ \angle ACB = 52.4°,\ AC = 20.4 \text{ cm} \ \text{ or } \ \angle ACB = 127.6°,\ AC = 5.4 \text{ cm} \ \text{(all 1dp)}$$

2.

a $\alpha = 180 - (40 + 32) = 108$

$$\frac{x}{\sin 108} = \frac{23.1}{\sin 40}$$

$$x = \frac{23.1 \times \sin 108}{\sin 40}$$

$x = 34.2$ cm (3sf)

b $x^2 = 2.7^2 + 3.8^2 - (2 \times 2.7 \times 3.8 \times \cos 83)$

$x^2 = 19.229$

$x = 4.39$ m (3sf)

c $\frac{\sin \alpha}{7.6} = \frac{\sin 61}{10.5}$

$$\sin \alpha = \frac{7.6 \times \sin 61}{10.5} = 0.6331$$

$\alpha = 39.276$

$\beta = 180 - (61 + 39.276) = 79.724$

$$\frac{x}{\sin 79.724} = \frac{10.5}{\sin 61}$$

$$x = \frac{10.5 \times \sin 79.724}{\sin 61}$$

$x = 11.8$ cm (3sf)

3.

a $\frac{\sin \alpha}{67} = \frac{\sin 96.5}{92}$

$$\sin \alpha = \frac{67 \times \sin 96.5}{92}$$

$\sin \alpha = 0.7236$

$\alpha = 46.351$

$\theta = 180 - 96.5 - \alpha$

$\theta = 37.1°$ (1dp)

b $1.9^2 = 0.8^2 + 1.7^2 - (2 \times 0.8 \times 1.7 \times \cos \theta)$

$$\cos \theta = \frac{0.8^2 + 1.7^2 - 1.9^2}{2 \times 0.8 \times 1.7}$$

$\cos \theta = -0.02941$

$\theta = 91.7°$ (1dp)

c $l^2 = 7.4^2 + 8.7^2 - (2 \times 7.4 \times 8.7 \times \cos 43.7)$

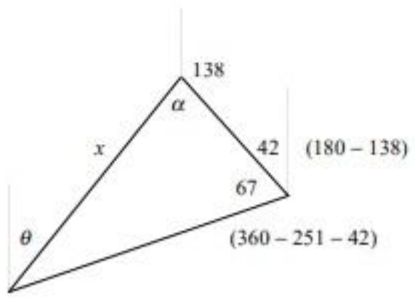
$l^2 = 37.3608,\ l = 6.1123$

$$\frac{\sin \theta}{7.4} = \frac{\sin 43.7}{6.1123}$$

$$\sin \theta = \frac{7.4 \times \sin 43.7}{6.1123} = 0.8364$$

$\theta = 56.8°$ (1dp)

4.



a $x^2 = 4.2^2 + 7.8^2 - (2 \times 4.2 \times 7.8 \times \cos 67)$

$x^2 = 52.879$

$x = 7.27$ miles (3sf)

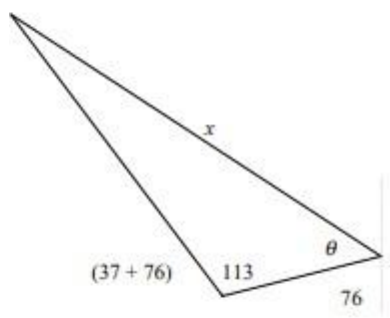
b $\frac{\sin \alpha}{7.8} = \frac{\sin 67}{7.2718}$

$$\sin \alpha = \frac{7.8 \times \sin 67}{7.2718} = 0.9874$$

$\alpha = 80.882$

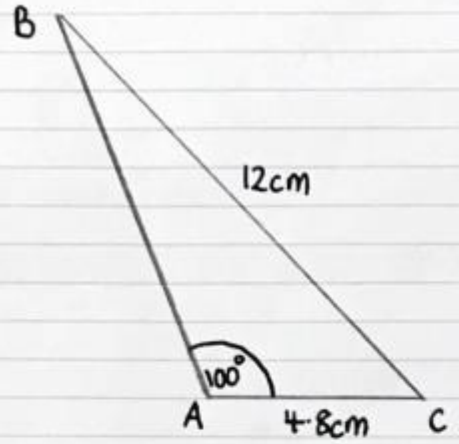
$\theta = 138 + \alpha - 180 = 38.882$

bearing = 039° (nearest degree)

5.	 $x^2 = 3.2^2 + 6.9^2 - (2 \times 3.2 \times 6.9 \times \cos 113)$ $x^2 = 75.105$ $x = 8.67$ km (3sf) $\frac{\sin\theta}{6.9} = \frac{\sin 113}{8.666}$ $\sin\theta = \frac{6.9 \times \sin 113}{8.666} = 0.7329$ $\theta = 47.130$ bearing $= 180 + 76 + \theta = 303°$ (nearest degree)
6.	$9.7^2 = 10.4^2 + 11.0^2 - (2 \times 10.4 \times 11.0 \times \cos \angle BAC)$ $\cos \angle BAC = \frac{10.4^2 + 11.0^2 - 9.7^2}{2 \times 10.4 \times 11.0} = 0.5903$ $\angle BAC = 53.819$ area $= \frac{1}{2} \times 10.4 \times 11.0 \times \sin 53.819 = 46.2$ cm^2
7.	$\frac{1}{2} \times 22.5 \times YZ \times \sin 34 = 100$ $YZ = \frac{200}{22.5 \times \sin 34} = 15.896$ $XZ^2 = 22.5^2 + 15.896^2 - (2 \times 22.5 \times 15.896 \times \cos 34)$ $= 165.906$ $XZ = 12.9$ cm (3sf)

Triangle Geometry Exam Questions

1. June 2006 Q2



(a) $\frac{\sin A\hat{B}C}{4.8} = \frac{\sin 100}{12}$ (M1)

$\sin A\hat{B}C = 0.4 \sin 100$ (M1)

$A\hat{B}C = \sin^{-1}(0.4 \sin 100)$

$A\hat{B}C = 23.1988\,2755$

$A\hat{B}C = 23.2$ correct to nearest $0.1°$ (A1)

(b) angle $A\hat{C}B = 180 - 100 - 23.2$

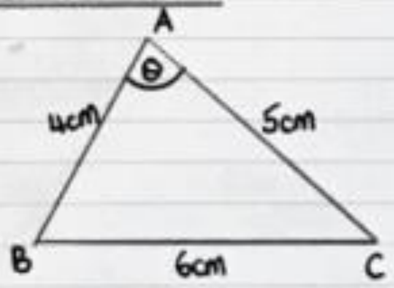
$= 56.8°$ (M1)

area of $\Delta = \frac{1}{2} \times 4.8 \times 12 \times \sin 56.8$ (M1)

$= 24.1$ cm^2 to 3.s.f. (A1)

/6

2. Jan 2007 Q4



(a) Using cosine rule (M1)

$6^2 = 4^2 + 5^2 - 2(4)(5)\cos\theta$

$36 = 41 - 40\cos\theta$

$40\cos\theta = 5$ (M1)

$\cos\theta = \frac{5}{40}$

$\cos\theta = \frac{1}{8}$ (A1)

(c) $\theta = \cos^{-1}\left(\frac{1}{8}\right)$

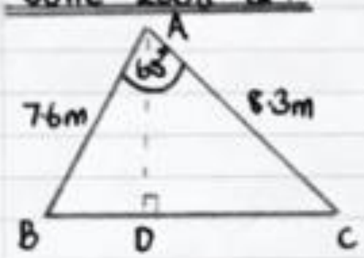
$\theta = 82.8°$ (M1)

$A = \frac{1}{2} \times 4 \times 5 \times \sin 82.8°$

Area $= 9.92\text{cm}^2$ to 3.s.f. (A1)

/5

3. June 2008 Q4



(a) Using cosine rule

$BC^2 = 7.6^2 + 8.3^2 - 2(7.6)(8.3)\cos 65°$ (M1)

$BC^2 = 73.33248...$ (M1)

$BC = 8.5634...$

$BC = 8.56\text{m}$ to 3.s.f. (A1)

(b) $A = \frac{1}{2} \times 7.6 \times 8.3 \sin 65$ (M1)

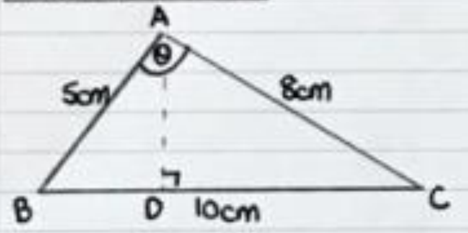
$= 28.6\text{m}^2$ to 3.s.f. (A1)

(c) $28.6 = \frac{1}{2} \times 8.56 \times AD$ (M1)(M1)

$AD = 6.7\text{m}$ to nearest 0.1m (A1)

/8

4. Jan 2011 Q3



(a) Using cosine rule

$10^2 = 5^2 + 8^2 - 2(5)(8)\cos\theta$ (M1)

$100 = 89 - 80\cos\theta$ (M1)

$80\cos\theta = -11$

$\cos\theta = -\frac{11}{80}$

$\theta = 97.9032...$

$\theta = 97.9°$ to nearest 0.1° (A1)

(b) (i) Area $= \frac{1}{2} \times 5 \times 8 \times \sin 97.9°$ (M1)

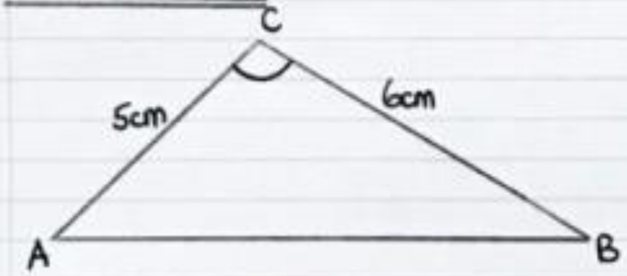
$= 19.8\text{cm}^2$ to 3.s.f. (A1)

(ii) $19.8 = \frac{1}{2} \times 10 \times AD$ (M1)(M1)

$AD = 3.96$ cm to 3.s.f. (A1)

/8

5. Jan 2013 Q3



(a) $12.5 = \frac{1}{2} \times 5 \times 6 \times \sin C$ (M1)

$\frac{12.5}{15} = \sin C$ (A1)

$C = 56.4°$

but $A\hat{C}B$ is obtuse

$\therefore A\hat{C}B = 180 - 56.4$

$= 123.6°$ (A1)

(b) Using cosine rule

$AB^2 = 5^2 + 6^2 - 2(5)(6)\cos 123.6°$ (M1)(M1)

$AB^2 = 94.203...$

$AB = 9.7$ cm to 2.s.f. (A1)

/6

TASK 2

Year 12 Initial Test for Mathematics

Write out the solutions to each of the following questions.
Show full working, ***without*** *the use of a calculator.*

Your initial test for mathematics will look exactly like this so use the videos and worksheets to ensure you are able to do the following skills with the layout expected.

Practice 1 (No Calculator)

B1 Indices

1.	Evaluate $\left(\frac{8}{125}\right)^{-2/3}$	**2.**	Express in the form x^k $\frac{\sqrt{x} \times \sqrt[3]{x}}{x^2}$	**3.**	Solve $9^{x-2} = 27$	**4.**	Solve $16^x = 4^{1-x}$

B2 Surds

1.	Simplify $\sqrt{72}$	**2.**	Expand and simplify $(2\sqrt{7} - 5\sqrt{3})\,(3\sqrt{7} + 4\sqrt{3})$	**3.**	Rationalise the denominator $\frac{11}{2\sqrt{5}}$	**4.**	Rationalise the denominator $\frac{8 - 3\sqrt{5}}{2 + \sqrt{5}}$

B3 Quadratics

1. Solve the following quadratic equations by factorising and use your solutions to sketch the related quadratic graph, labelling all intersections with the coordinate axis.

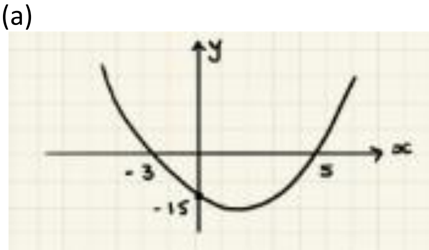
(a) (i) $x^2 + 3x - 28 = 0$	(b) (i) $x^2 - 6x + 9 = 0$	(c) (i) $2x^2 - 21x + 27 = 0$
(a) (ii) Sketch $y = x^2 + 3x - 28$	(b) (ii) Sketch $y = x^2 - 6x + 9$	(c) (ii) Sketch $y = 2x^2 - 21x + 27$

2. Solve the following quadratic equations by completing the square and use your solutions to sketch the related quadratic graph, labelling all intersections with the coordinate axis and turning point.

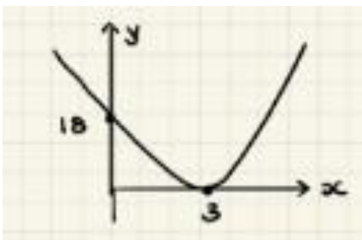
(a) (i) $x^2 + 4x - 7 = 0$	(b) (i) $11 + 8x - x^2 = 0$	(c) (i) $3x^2 - 12x + 2 = 0$
(ii) Write $y = x^2 + 4x - 7$ in the form $y = a(x + b)^2 + c$	(ii) Write $y = 11 + 8x - x^2$in the form $y = a(x + b)^2 + c$	(ii) Write $y = 3x^2 - 12x + 2$ in the form $y = a(x + b)^2 + c$
(iii) Sketch $y = x^2 + 4x - 7$	(iii) Sketch $y = 11 + 8x - x^2$	(iii) Sketch $y = 3x^2 - 12x + 2$

3. Evaluate the equation of the following quadratics, giving your answer in the form $y = ax^2 + bx + c$

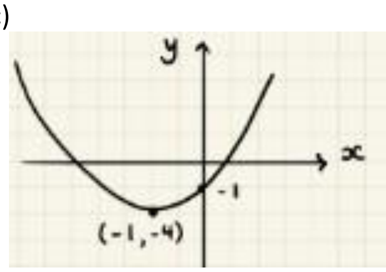
(a)



(b)



(c)



B4 Simultaneous Equations

1.	Solve $3x + 3y = -4$ $5x - 2y = 5$	**2.**	Solve $y = x - 6$ $\frac{1}{2}x - y = 4$	**3.**	Solve $3x^2 - x - y^2 = 0$ $x + y = 1$

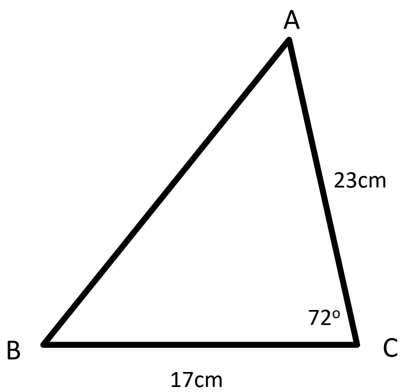
B5 Inequalities

Find the set of values for which…

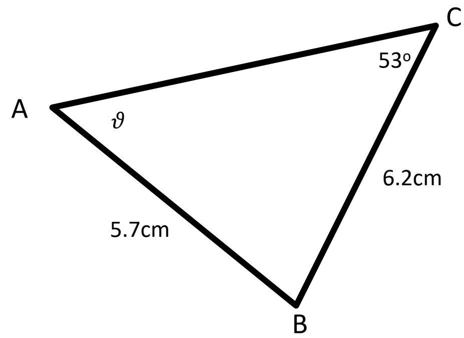
1.	$3(1 - 2t) \leq t - 4$	**2.**	$2x^2 - 9x + 4 \leq 0$	**3.**	$2y + 3 < 3y(y - 2)$

E1 Triangle Geometry (Calculator)

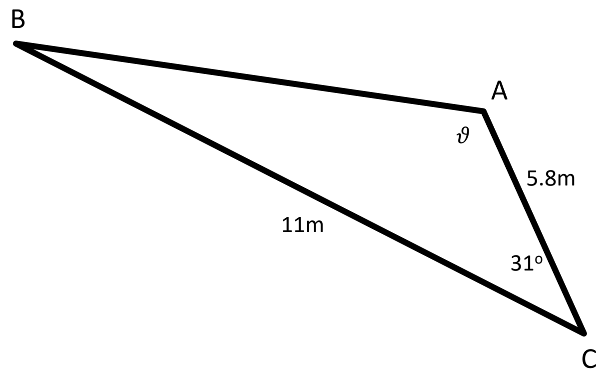
1. Calculate the length AB



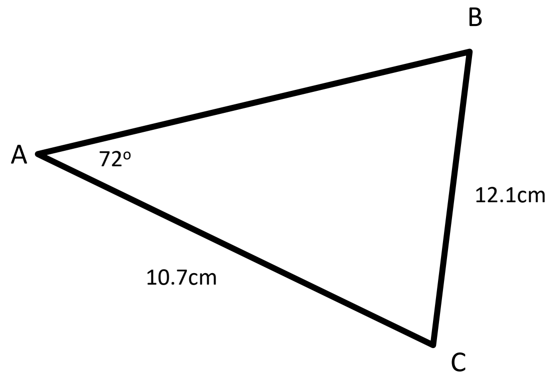
2. Calculate the angle ϑ



3. Calculate the length AB and the obtuse angle ϑ



4. Calculate the area of the triangle ABC



Practice 1

B1 Indices

1. $\left(\frac{8}{125}\right)^{-2/3}$

$= \left(\frac{125}{8}\right)^{2/3}$

$= \left(\frac{5}{2}\right)^{2}$ M1

$= \frac{25}{4}$ A1

2. $\frac{\sqrt{x} \times \sqrt[3]{x}}{x^2}$

$= \frac{x^{1/2} \times x^{1/3}}{x^2}$ M1

$= \frac{x^{5/6}}{x^2}$ A1

$= x^{-7/6}$ A1

3. $9^{x-2} = 27$

$(3^2)^{x-2} = 3^3$ M1

$3^{2x-4} = 3^3$

$2x - 4 = 3$ M1

$2x = 7$

$x = 7/2$ A1

4. $16^{x} = 4^{1-x}$

$(4^2)^{x} = 4^{1-x}$ M1

$4^{2x} = 4^{1-x}$

$2x = 1 - x$ M1

$3x = 1 \Rightarrow x = 1/3$ A1

11

B2 Surds

1. $\sqrt{72}$

$= \sqrt{36 \times 2}$

$= 6\sqrt{2}$ A1

2. $(2\sqrt{7} - 5\sqrt{3})(3\sqrt{7} + 4\sqrt{3})$

$42 + 8\sqrt{21} - 15\sqrt{21} - 60$ M1 A1

$-7\sqrt{21} - 18$ A1

3. $\frac{11}{2\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$ M1

$= \frac{11\sqrt{5}}{10}$ A1

4. $\frac{8 - 3\sqrt{5}}{2 + \sqrt{5}} \; \frac{(2 - \sqrt{5})}{(2 - \sqrt{5})}$ M1

$= \frac{16 - 8\sqrt{5} - 6\sqrt{5} + 15}{4 - 5}$ A1 A1

$= \frac{31 - 14\sqrt{5}}{-1} = 14\sqrt{5} - 31$ A1

10

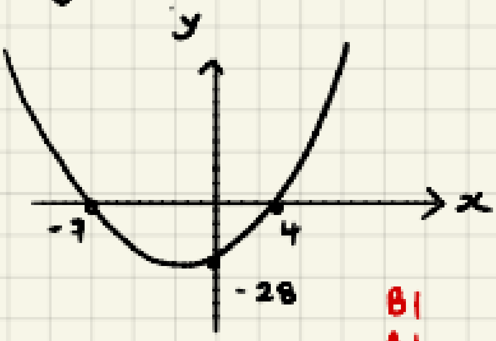
B3 Quadratics

1. (a) (i) $x^2 + 3x - 28 = 0$

$(x+7)(x-4) = 0$ M1

$x = -7$ or $x = 4$ A1

(ii) $y = x^2 + 3x - 28$



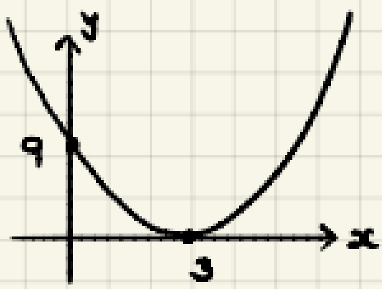
B1
A1
A1

(b) (i) $x^2 - 6x + 9 = 0$

$(x-3)^2 = 0$ M1

A1 $x = 3$ (repeated)

(ii) $y = x^2 - 6x + 9$



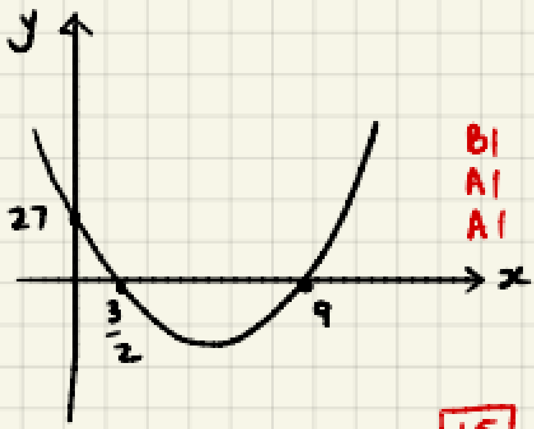
B1
A1
A1

(c) (i) $2x^2 - 21x + 27 = 0$

$(2x-3)(x-9) = 0$ M1

$x = 3/2$ $x = 9$ A1

(ii) $y = 2x^2 - 21x + 27$



B1
A1
A1

15

B1 shape, location related to axes

A1 intersections x-axis

A1 intersections y-axis

2. (a) (i) $x^2 + 4x - 7 = 0$

$(x+2)^2 - 4 - 7 = 0$ M1

$(x+2)^2 = 11$

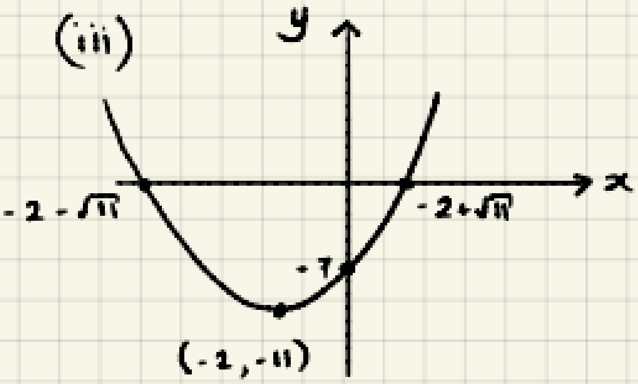
$x + 2 = \pm\sqrt{11}$

$x = -2 \pm \sqrt{11}$ A1

(ii) $y = x^2 + 4x - 7$

$y = (x+2)^2 - 11$ B1

(iii)



Graphs
- B1 Shape
- A1 Vertex
- A1 intersections x-axis
- A1 intersections y-axis

(b) (i) $11 + 8x - x^2 = 0$

$-(x^2 - 8x - 11) = 0$ M1

$-[(x-4)^2 - 16 - 11] = 0$ M1

$-(x-4)^2 + 27 = 0$

$(x-4)^2 = 27$

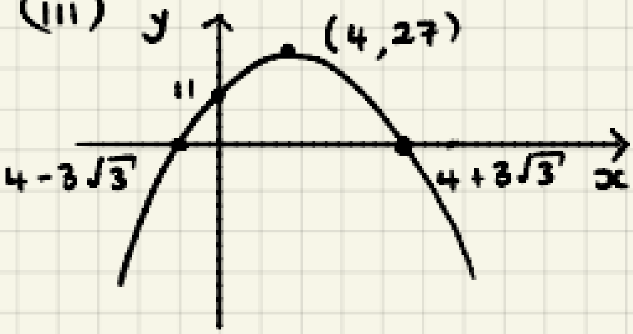
$x - 4 = \pm 3\sqrt{3}$

$x = 4 \pm 3\sqrt{3}$ A1

(ii) $y = 11 + 8x - x^2$

$y = 27 - (x-4)^2$ B1

(iii)



(c) (i) $3x^2 - 12x + 2 = 0$

$3\left[x^2 - 4x + \frac{2}{3}\right] = 0$ M1

$3\left[(x-2)^2 - 4 + \frac{2}{3}\right] = 0$ M1

$3\left[(x-2)^2 - \frac{10}{3}\right] = 0$

$3(x-2)^2 - 10 = 0$

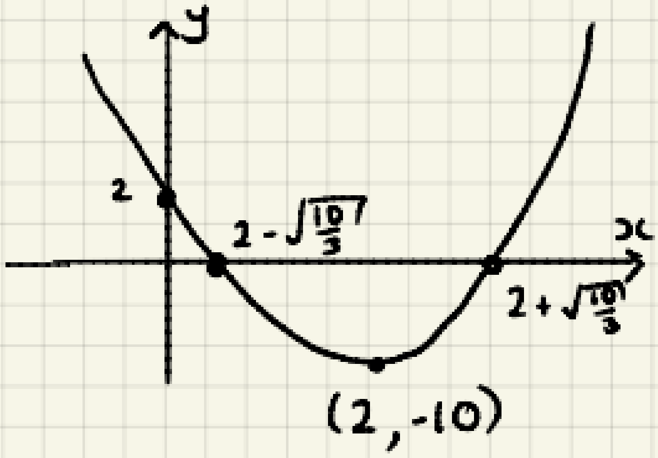
$(x-2)^2 = \frac{10}{3}$

$x - 2 = \pm\sqrt{\frac{10}{3}}$

$x = 2 \pm \sqrt{\frac{10}{3}}$ A1

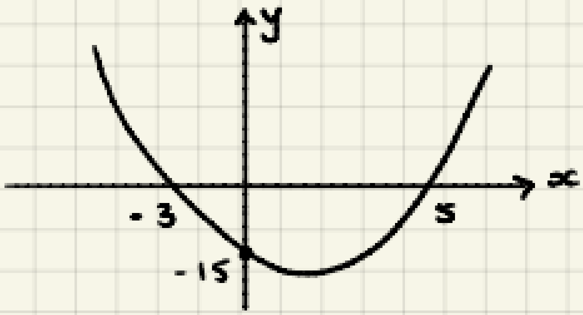
(ii) $y = 3x^2 - 12x + 2$

$y = 3(x-2)^2 - 10$ B1



23

3. (a)



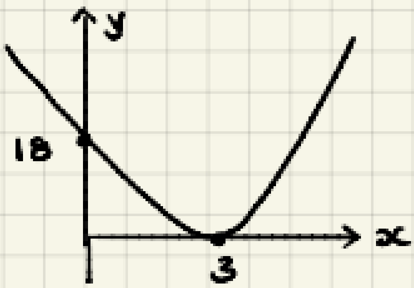
$y = k(x+3)(x-5)$ M1

$-15 = k(3)(-5) \Rightarrow k = 1$ A1

$y = (x+3)(x-5)$

$y = x^2 - 2x - 15$ A1

(b)



$y = k(x-3)^2$ M1

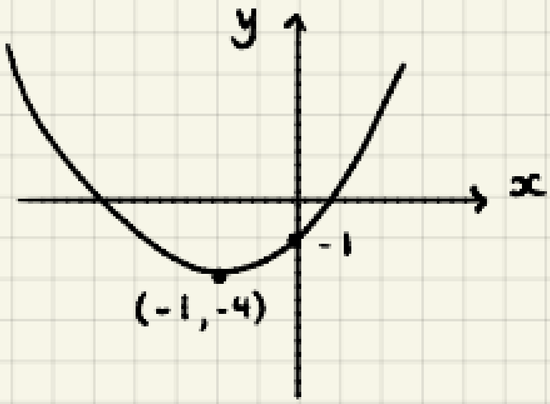
$18 = k(-3)^2 \Rightarrow k = 2$ A1

$y = 2(x-3)^2$

$y = 2(x^2 - 6x + 9)$

$y = 2x^2 - 12x + 18$ A1

(c)



$y = k(x+1)^2 - 4$ M1

$(0,-1)$

$-1 = k(1)^2 - 4$

$\Rightarrow k = 3$ A1

$y = 3(x+1)^2 - 4$

$y = 3(x^2 + 2x + 1) - 4$

$y = 3x^2 + 6x - 1$ A1

[9]

84 Simultaneous Equations

1. $3x + 3y = -4$ $\quad$ $6x + 6y = -8$

$5x - 2y = 5$ $\quad$ $15x - 6y = 15$ add $\quad$ M1

$21x = 7$

$x = 1/3$ A1 $\quad$ $3(1/3) + 3y = -4$

$3y = -5$

$x = 1/3$, $y = -5/3$ A1

2. $y = x - 6$

$\frac{1}{2}x - y = 4$

$\frac{1}{2}x - (x - 6) = 4$ $\quad$ M1

$\frac{1}{2}x - x + 6 = 4$

$-\frac{1}{2}x = -2$

$x = 4$ A1 $\quad$ $y = 4 - 6$

$y = -2$

$x = 4$, $y = -2$ A1

3. $3x^2 - x - y^2 = 0 \qquad x + y = 1$

$3x^2 - x - (1-x)^2 = 0$ M1 $\qquad y = 1 - x$

$3x^2 - x - (1 - 2x + x^2) = 0$

$3x^2 - x - 1 + 2x - x^2 = 0$

$2x^2 + x - 1 = 0$ A1

$(2x-1)(x+1) = 0$

$x = 1/2 \qquad x = -1$ A1

$y = 1 - 1/2 \qquad y = 1 - -1$

$x = 1/2 \; y = 1/2$ A1 $\qquad x = -1, y = 2$ A1

[11]

B5 Inequalities

1. $3(1-2t) \leq t - 4$

$3 - 6t \leq t - 4$

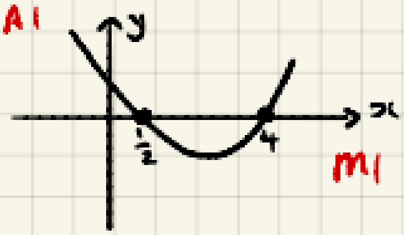
$7 \leq 7t$ M1

$t \geq 1$ A1

2. $2x^2 - 9x + 4 \leq 0$

$(2x-1)(x-4) \leq 0$ M1

CVs $x = 1/2 \quad x = 4$ A1



M1

$\frac{1}{2} \leq x \leq 4$ A1

3. $2y + 3 < 3y(y-2)$

$2y + 3 < 3y^2 - 6y$

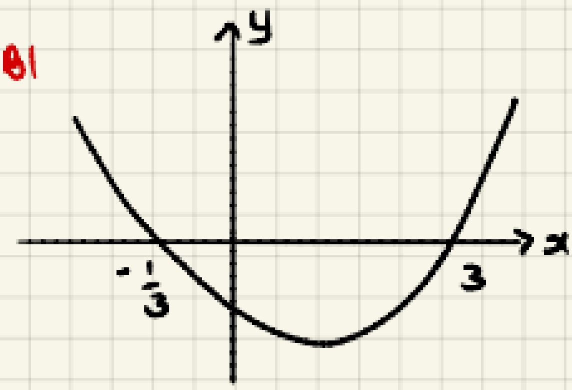
$0 < 3y^2 - 8y - 3$ B1

$3y^2 - 8y - 3 > 0$

$(3y+1)(y-3) > 0$ M1

CVs $y = -1/3 \quad y = 3$ A1

[11]

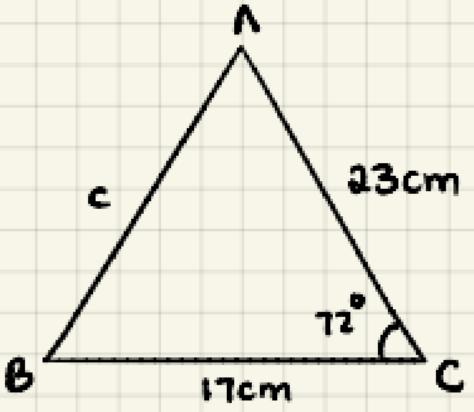


M1

$y < -1/3 \quad$ or $\quad y > 3$ A1

E1 Triangle Geometry

1.



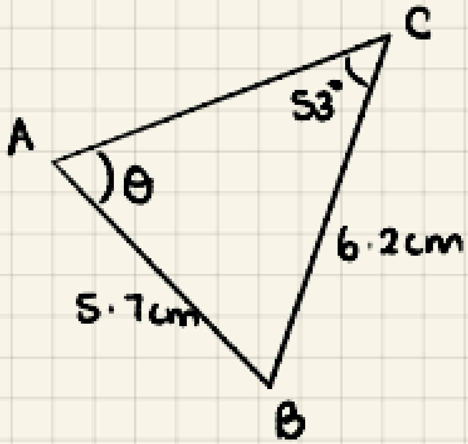
$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 17^2 + 23^2 - 2(17)(23)\cos 72°$ M1

$c^2 = 576.35$

$AB = 24.0\text{cm}$ A1

2.

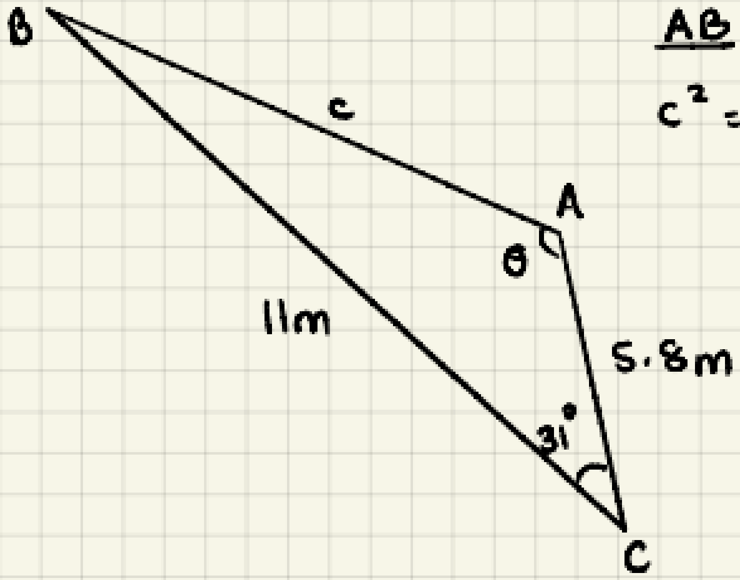


$\frac{\sin\theta}{6.2} = \frac{\sin 53}{5.7}$ M1

$\theta = \sin^{-1}\left(\frac{6.2\sin 53}{5.7}\right)$

$\theta = 60.3°$ A1

3.



AB

$c^2 = 5.8^2 + 11^2 - 2(5.8)(11)\cos 31$

$c^2 = 45.27$ M1

$AB = 6.7\text{m}$ A1

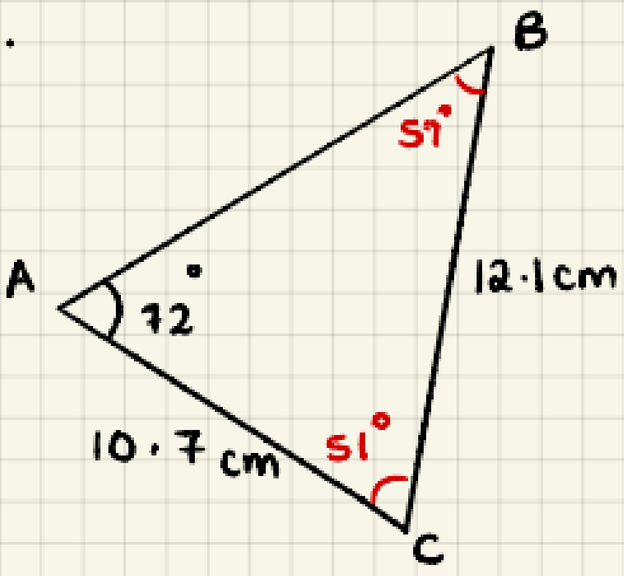
θ

$\cos\theta = \frac{5.8^2 + 6.7^2 - 11^2}{2(5.8)(6.7)}$ M1

$\theta = \cos^{-1}(-0.546)$ A1

$\theta = 123°$

4.



$$\frac{\sin B}{10.7} = \frac{\sin 72°}{12.1}$$

M1

$$B = \sin^{-1}\left(\frac{10.7 \sin 72°}{12.1}\right)$$

$\theta = 57°$ A1

$$A = \frac{1}{2}ab\sin C$$

M1

$$= \frac{1}{2}(10.7)(12.1)\sin 51°$$

$= 50.3\text{cm}^2$ A1

[12]

Year 12 Initial Test for Mathematics

Write out the solutions to each of the following questions.
Show full working, ***without*** *the use of a calculator.*

Your initial test for mathematics will look exactly like this so use the videos and worksheets to ensure you are able to do the following skills with the layout expected.

Practice 2 (No Calculator)

B1 Indices

1.	Evaluate $\left(3\frac{3}{8}\right)^{-1/3}$	**2.**	Express in the form x^k $\frac{\sqrt{x}\times\sqrt[5]{x}}{x^2}$	**3.**	Solve $3^{3x-2}=\sqrt[3]{9}$	**4.**	Solve $\left(\frac{1}{2}\right)^{1-x}=\left(\frac{1}{8}\right)^{2x}$

B2 Surds

1.	Simplify $\sqrt{80}$	**2.**	Expand and simplify $(7-3\sqrt{5})\,(3\sqrt{5}-2)$	**3.**	Rationalise the denominator $\frac{7}{5\sqrt{3}}$	**4.**	Rationalise the denominator $\frac{3+5\sqrt{11}}{7-\sqrt{11}}$

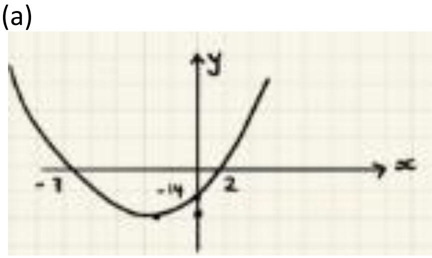
B3 Quadratics

1. Solve the following quadratic equations by factorising and use your solutions to sketch the related quadratic graph, labelling all intersections with the coordinate axis.		
(a) (i) $x^2-13x+40=0$	(b) (i) $x^2+5x=0$	(c) (i) $6x^2+5x-4=0$
(a) (ii) Sketch $y=x^2-13x+40$	(b) (ii) Sketch $y=x^2+5x$	(c) (ii) Sketch $y=6x^2+5x-4$

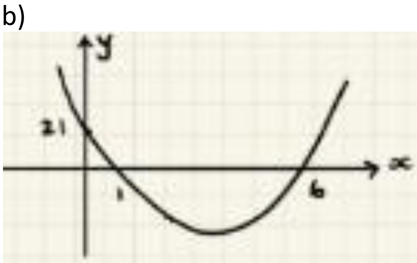
2. Solve the following quadratic equations by completing the square and use your solutions to sketch the related quadratic graph, labelling all intersections with the coordinate axis and turning point.		
(a) (i) $x^2+2x-20=0$	(b) (i) $-11+8x-x^2=0$	(c) (i) $3x^2-18x+2=0$
(ii) Write $y=x^2+2x-20$ in the form $y=a(x+b)^2+c$	(ii) Write $y=-11+8x-x^2$in the form $y=a(x+b)^2+c$	(ii) Write $y=3x^2-18x+2$ in the form $y=a(x+b)^2+c$
(iii) Sketch $y=x^2+2x-20$	(iii) Sketch $y=-11+8x-x^2$	(iii) Sketch $y=3x^2-18x+2$

3. Evaluate the equation of the following quadratics, giving your answer in the form $y=ax^2+bx+c$

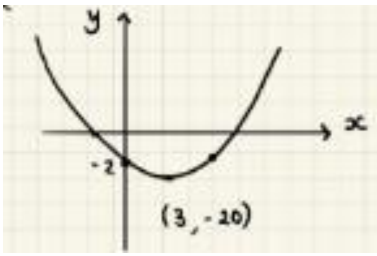
(a)



(b)



(c)



B4 Simultaneous Equations

1.	Solve $3x - 4y = 16$ $2x + 12y = 7$	**2.**	Solve $3y = 2x - 8$ $4x + y = -5$	**3.**	Solve $3x^2 - xy + y^2 = 36$ $x - 2y = 10$

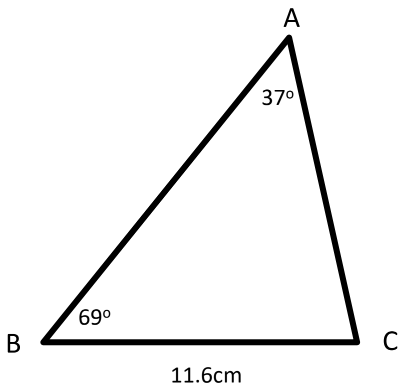
B5 Inequalities

Find the set of values for which…

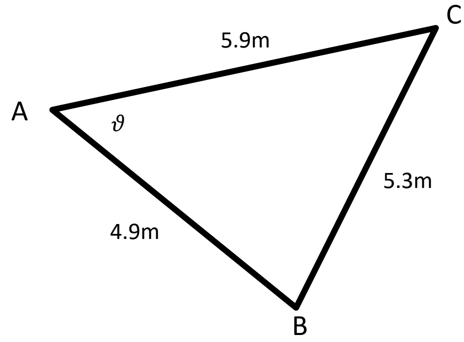
1.	$4(5 - 2y) \geq 3(7 - 2y)$	**2.**	$2x^2 - 5x - 3 > 0$	**3.**	$x(2x + 1) \leq x^2 + 6$

E1 Triangle Geometry (Calculator)

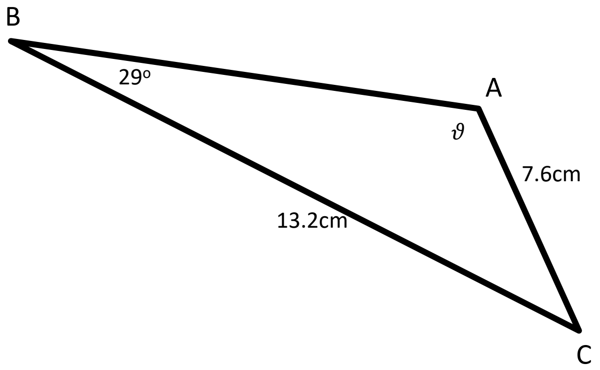
1. Calculate the length AB



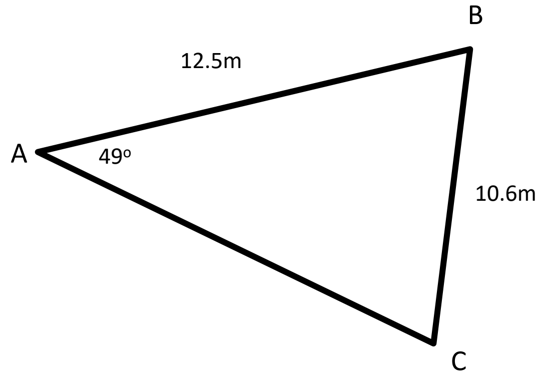
2. Calculate the angle ϑ



3. Calculate the length AB and the obtuse angle ϑ



4. Calculate the area of the triangle ABC



Practice Test 2

B1 Indices

1. $\left(3\frac{3}{8}\right)^{-1/3} = \left(\frac{27}{8}\right)^{-\frac{1}{3}}$ M1

$= \left(\frac{8}{27}\right)^{1/3}$

$= \frac{2}{3}$ A1

2. $\frac{\sqrt{x} \times \sqrt[5]{x}}{x^2}$

$= \frac{x^{1/2} \times x^{1/5}}{x^2}$ M1

$= \frac{x^{7/10}}{x^2}$ A1 $= x^{-13/10}$ A1

3. $3^{3x-2} = \sqrt[3]{9}$

$3^{3x-2} = (3^2)^{1/3}$ M1

$3^{3x-2} = 3^{2/3}$

$3x - 2 = \frac{2}{3}$ M1

$3x = \frac{8}{3} \Rightarrow x = \frac{8}{9}$ A1

4. $\left(\frac{1}{2}\right)^{1-x} = \left(\frac{1}{8}\right)^{2x}$

$(2^{-1})^{1-x} = (2^{-3})^{2x}$ M1

$2^{-1+x} = 2^{-6x}$

$-1 + x = -6x$ M1

$7x = 1$

$x = 1/7$ A1

[11]

B2 Surds

1. $\sqrt{80}$

$= \sqrt{16 \times 5}$

$= 4\sqrt{5}$ A1

2. $(7 - 3\sqrt{5})(3\sqrt{5} - 2)$

$= 21\sqrt{5} - 14 - 45 + 6\sqrt{5}$ M1 A1

$= 27\sqrt{5} - 59$ A1

3. $\frac{7}{5\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ M1

$= \frac{7\sqrt{3}}{15}$ A1

4. $\frac{3 + 5\sqrt{11}}{7 - \sqrt{11}} \cdot \frac{(7 + \sqrt{11})}{(7 + \sqrt{11})}$ M1

$= \frac{21 + 3\sqrt{11} + 35\sqrt{11} + 55}{49 - 11}$ A1 A1

$= \frac{76 + 38\sqrt{11}}{38}$

$= 2 + \sqrt{11}$ A1

[10]

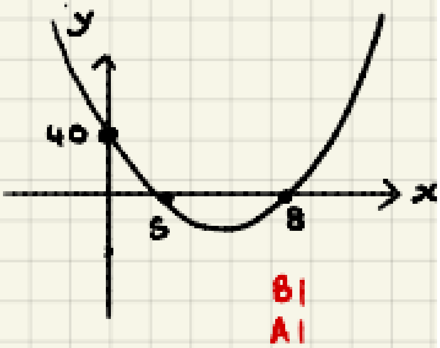
B3 Quadratics

1. (a) (i) $x^2 - 13x + 40 = 0$

$(x-8)(x-5) = 0$ M1

$x = 8 \quad x = 5$ A1

(ii) $y = x^2 - 13x + 40$



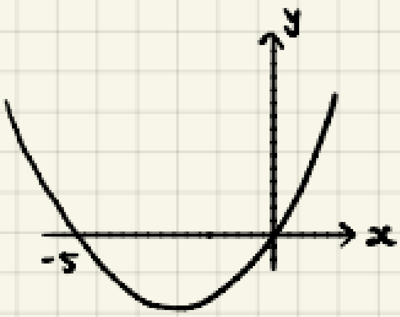
B1
A1
A1

(b) (i) $x^2 + 5x = 0$

$x(x+5) = 0$ M1

$x = 0 \quad x = -5$ A1

(ii) $y = x^2 + 5x$



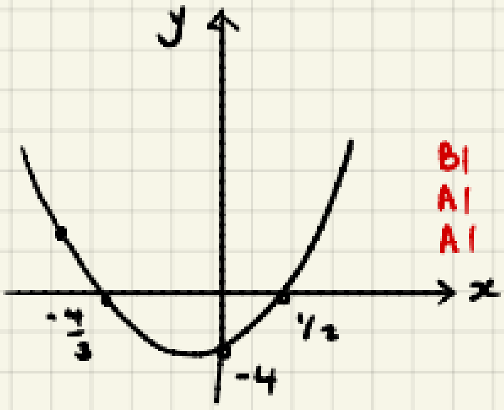
B1
A1
A1

(c) (i) $6x^2 + 5x - 4 = 0$

$(3x+4)(2x-1) = 0$ M1

$x = -4/3 \quad x = 1/2$ A1

(ii) $y =$



B1
A1
A1

15

B1 shape, location related to axes

A1 intersections x-axis

A1 intersections y-axis

2. (a) (i) $x^2 + 2x - 20 = 0$

$(x+1)^2 - 1 - 20 = 0$ M1

$(x+1)^2 = 21$

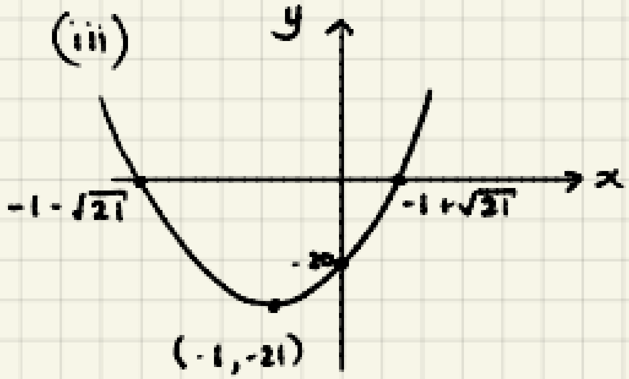
$x+1 = \pm\sqrt{21}$

$x = -1 \pm \sqrt{21}$ A1

(ii) $y = x^2 + 2x - 20$

$y = (x+1)^2 - 21$ B1

(iii)



Graphs

B1 Shape

A1 Vertex

A1 intersections x-axis

A1 intersections y-axis

(b) (i) $-11 + 8x - x^2 = 0$

$-(x^2 - 8x + 11) = 0$ M1

$-\left[(x-4)^2 - 16 + 11\right] = 0$ M1

$5 - (x-4)^2 = 0$

$(x-4)^2 = 5$

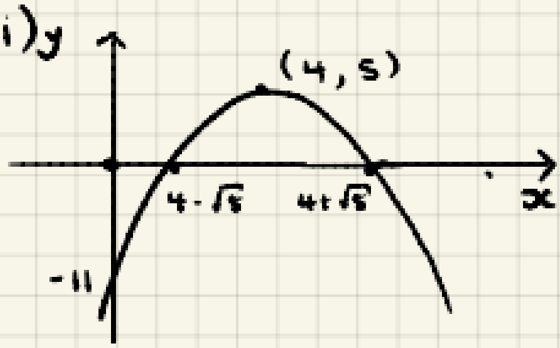
$x - 4 = \pm\sqrt{5}$

$x = 4 \pm \sqrt{5}$ A1

(ii) $y = -11 + 8x - x^2$

$y = 5 - (x-4)^2$ B1

(iii)



(c) (i) $3x^2 - 18x + 2 = 0$

$3\left[x^2 - 6x + \frac{2}{3}\right] = 0$ M1

$3\left[(x-3)^2 - 9 + \frac{2}{3}\right] = 0$ M1

$3\left[(x-3)^2 - \frac{25}{3}\right] = 0$

$3(x-3)^2 - 25 = 0$

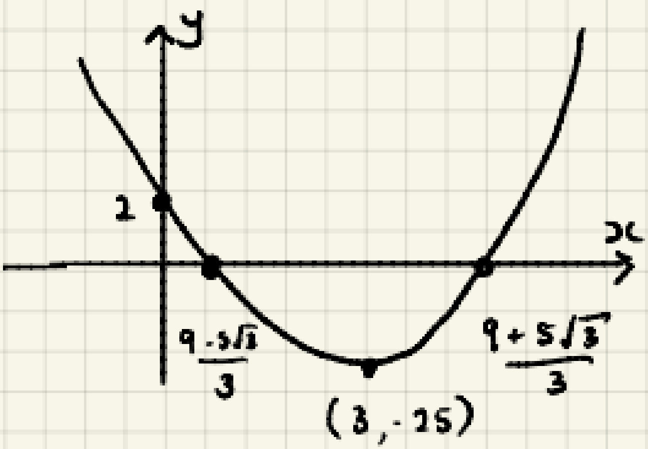
$3(x-3)^2 = 25$

$x - 3 = \pm\frac{5}{\sqrt{3}}$

$x = \frac{9 \pm 5\sqrt{3}}{3}$ A1

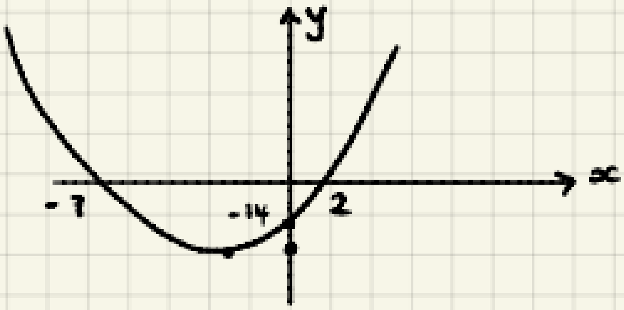
(ii) $y = 3x^2 - 18x + 2$

$y = 3(x-3)^2 - 25$ B1



23

3. (a)



$y = k(x+7)(x-2)$ M1

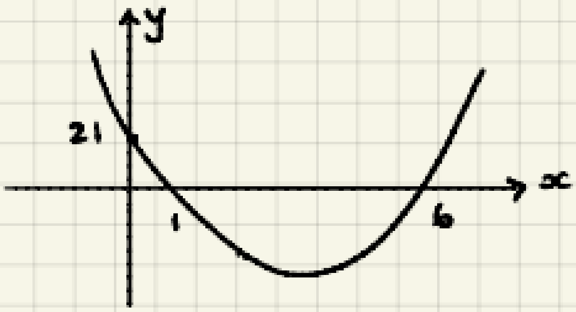
$-14 = k(7)(-2)$

$k = 1$ A1

$y = (x+7)(x-2)$

$y = x^2 + 5x - 14$ A1

(b)



$y = k(x-1)(x-6)$ M1

$21 = k(-1)(-6)$

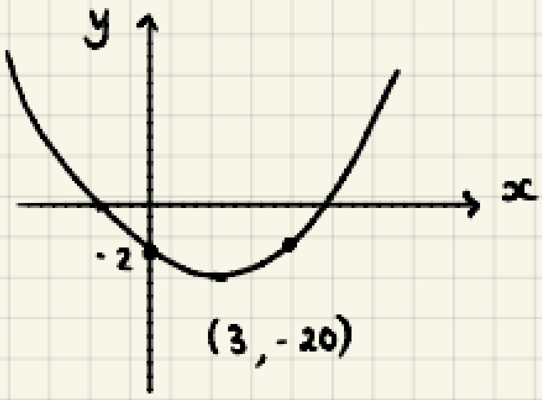
$\Rightarrow k = \frac{21}{6} = \frac{7}{2}$ A1

$y = \frac{7}{2}(x-1)(x-6)$

$y = \frac{7}{2}(x^2 - 7x + 6)$

$y = \frac{7x^2}{2} - \frac{49x}{2} + 21$ A1

(c)



$y = k(x-3)^2 - 20$ M1

$-2 = k(-3)^2 - 20$

$18 = k(9)$

$k = 2$ A1

$y = 2(x-3)^2 - 20$

$y = 2(x^2 - 6x + 9) - 20$

$y = 2x^2 - 12x - 2$ A1

[9]

B4. Simultaneous equations

1. $3x - 4y = 16$ $\qquad 9x - 12y = 48$

$2x + 12y = 7$ $\qquad 2x + 12y = 7$ M1

$11x = 55$

$x = 5$ A1

$3x - 4y = 16$
$15 - 4y = 16$
$-1 = 4y$
$y = -1/4$

$x = 5, y = -1/4$ A1

2. $3y = 2x - 8 \Rightarrow 2x = 3y + 8$
$4x = 6y + 16$

$4x + y = -5$ M1

$6y + 16 + y = -5$

$7y = -21$

$y = -3$ $\qquad 2x = 3y + 8$
$2x = 3(-3) + 8$
$x = -1/2$ A1 $\qquad x = -1/2 \quad y = -3$ A1

3. $3x^2 - xy + y^2 = 36$

$x - 2y = 10 \Rightarrow x = 2y + 10$

$3(2y+10)^2 - (2y+10)y + y^2 = 36$ M1

$3(4y^2 + 40y + 100) - y(2y+10) + y^2 = 36$

$12y^2 + 120y + 300 - 2y^2 - 10y + y^2 = 36$

$11y^2 + 110y + 264 = 0$

$y^2 + 10y + 24 = 0$ A1

$(y+6)(y+4) = 0$ M1

$y = -6 \qquad y = -4$

$x = 2(-6) + 10 \qquad x = 2(-4) + 10$

$x = -2 \qquad x = 2$

$x = -2, y = -6$ A1 $\qquad x = 2, y = -4$ A1

11

B5 Inequalities

1. $4(5-2y) \geq 3(7-2y)$

$20 - 8y \geq 21 - 6y$ M1

$-1 \geq 2y$

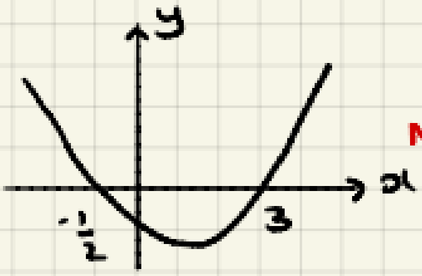
$-1/2 \geq y$

$y \leq -1/2$ A1

2. $2x^2 - 5x - 3 > 0$

$(2x+1)(x-3) > 0$ M1

CVs $x = -1/2$ $x = 3$ A1



M1

$x < -1/2$ or $x > 3$ A1

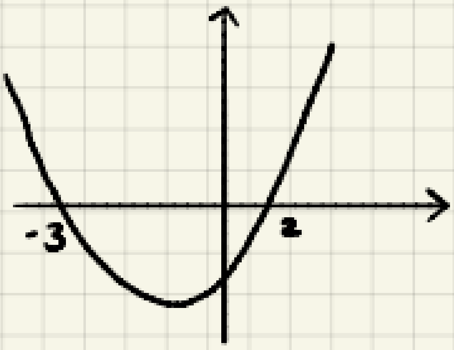
3. $x(2x+1) \leq x^2 + 6$

$2x^2 + x \leq x^2 + 6$ M1

$x^2 + x - 6 \leq 0$

$(x+3)(x-2) \leq 0$ M1

CVs $x = -3$ $x = 2$ A1



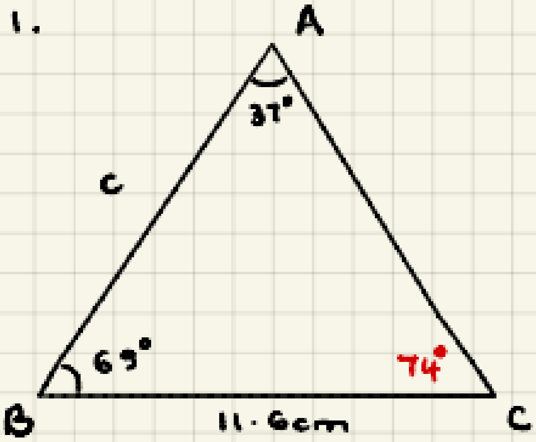
M1

$-3 \leq x \leq 2$ A1

[11]

E1 Triangle Geometry

1.

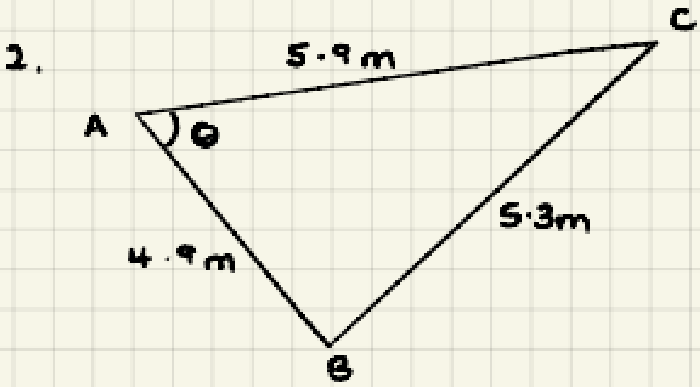


$$\frac{c}{\sin 74^\circ} = \frac{11.6}{\sin 37^\circ}$$

$$c = \frac{11.6 \sin 74^\circ}{\sin 37^\circ} \quad \text{M1}$$

$$c = 18.5\text{cm} \quad \text{A1}$$

2.



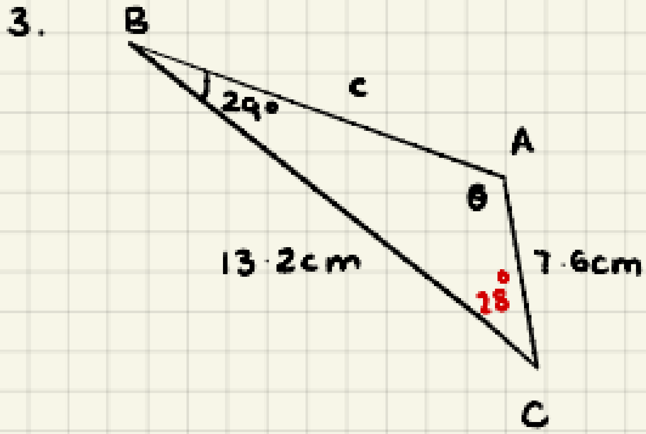
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos\theta = \frac{5.9^2 + 4.9^2 - 5.3^2}{2(5.9)(4.9)} \quad \text{M1}$$

$$\cos\theta = 0.53148$$

$$\theta = 57.9^\circ \quad \text{A1}$$

3.



θ

$$\frac{\sin\theta}{13.2} = \frac{\sin 29}{7.6}$$

$$\sin\theta = \frac{13.2 \sin 29^\circ}{7.6} \quad \text{M1}$$

$$\sin\theta = 0.8420$$

$$\theta = 57.4^\circ$$

obtuse $\Rightarrow \theta = 123^\circ$ A1

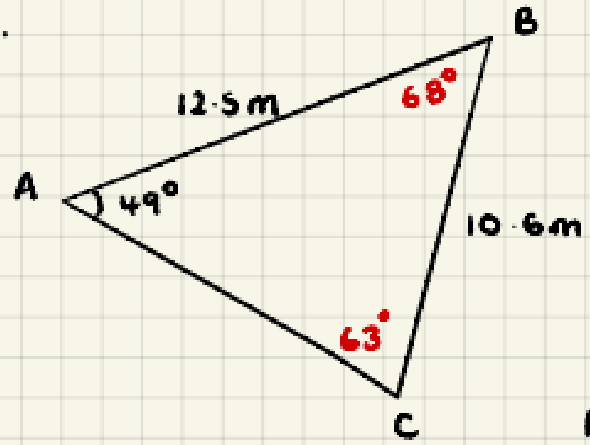
AB

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$c^2 = 13.2^2 + 7.6^2 - 2(13.2)(7.6)\cos 28 \quad \text{M1}$$

$$c^2 = 54.8 \Rightarrow c = 7.4\text{ cm} \quad \text{A1}$$

4.



$$\frac{\sin C}{12 \cdot 5} = \frac{\sin 49°}{10 \cdot 6}$$

$$\sin C = \frac{12 \cdot 5 \sin 49}{10 \cdot 6}$$ M1

$$C = 63°$$ A1

$$\text{Area} = \tfrac{1}{2} ab \sin C$$

$$\text{Area} = \tfrac{1}{2}(12 \cdot 5)(10 \cdot 6) \sin 68°$$ M1

$$\text{Area} = 61 \cdot 5 \text{ m}^2$$ A1

[2]

TASK 3

2 Trigonometry

The following two aspects are worth emphasising at this stage.

2.1 Trigonometric Equations

You can of course get one solution to an equation such as $\sin x = -0.5$ from your calculator. But what about others?

Example 1 Solve the equation $\sin x° = -0.5$ for $0 \le x < 360$.

Solution The calculator gives $\sin^{-1}(0.5) = -30$.

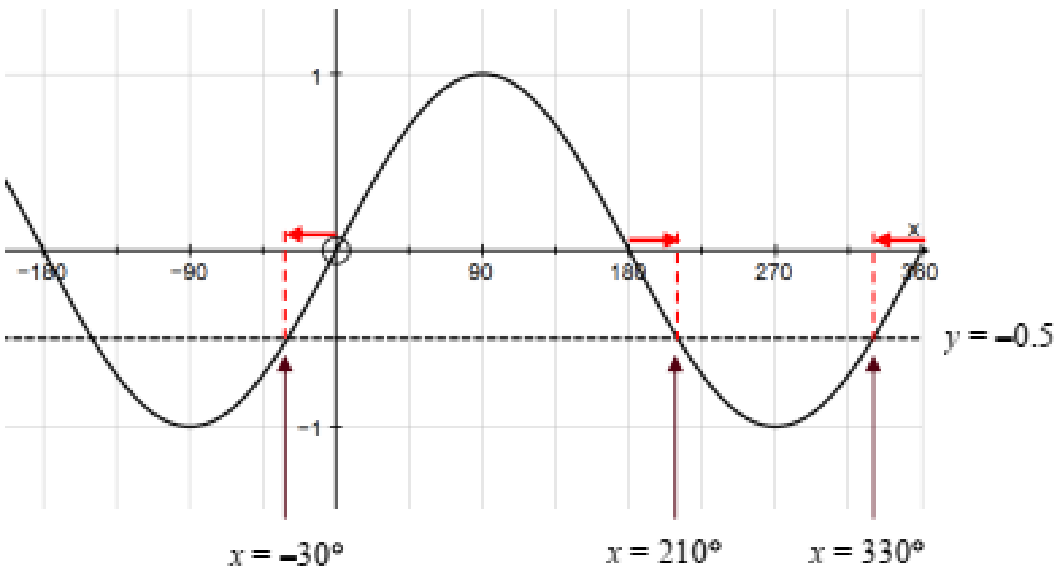
This is usually called the *principal value* of the function $\sin^{-1}$.

To get a second solution you can either use a graph or a standard rule.

Method 1: Use the graph of $y = \sin x$

By drawing the line $y = -0.5$ on the same set of axes as the graph of the sine curve, points of intersection can be identified in the range

$0 \le x < 360$.



(The red arrows each indicate 30° to one side or the other.)

Hence the required solutions are 210° or 330°.

Method 2: Use an algebraic rule.

To find the second solution you use

$$\sin(180 - x)° = \sin x°$$

$$\tan(180 + x)° = \tan x°$$

$$\cos(360 - x)° = \cos x°.$$

Any further solutions are obtained by adding or subtracting 360 from the principal value or the second solution.

In this example the principal solution is $-30°$.

Therefore, as this equation involves sine, the second solution is:

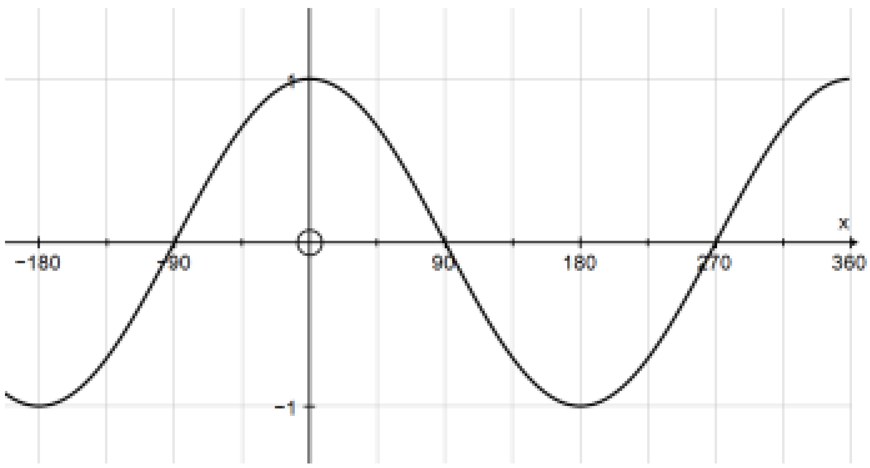
$$180 - (-30)° = 210°$$

$-30°$ is not in the required range, so add 360 to get:

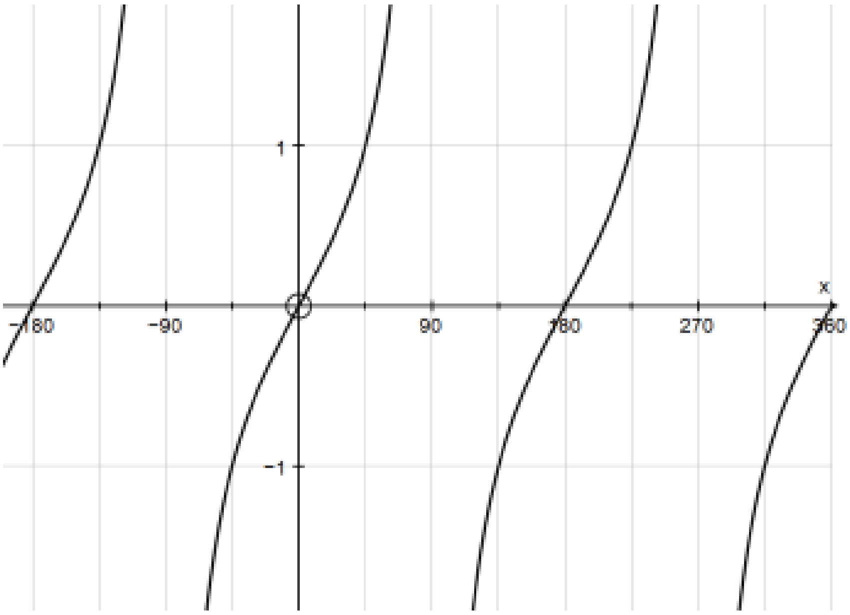
$$360 + (-30) = 330°.$$

Hence the required solutions are $210°$ or $330°$.

You should decide which method you prefer. The corresponding graphs for $\cos x$ and $\tan x$ are shown below.



$y = \cos x$



$y = \tan x$

To solve equations of the form $y = \sin(kx)$, you will expect to get $2k$ solutions in any interval of 360°. You can think of compressing the graphs, or of using a wider initial range.

Example 2 Solve the equation $\sin 3x° = 0.5$ for $0 \le x < 360$.

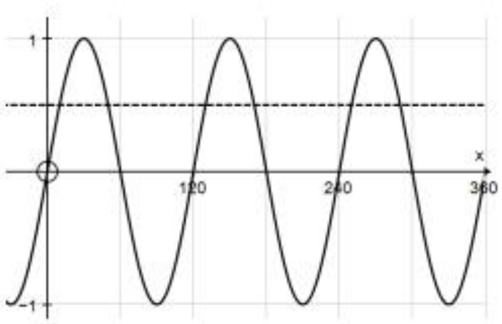
Solution ***Method 1:*** Use the graph.

The graph of $y = \sin 3x°$ is the same as the graph of $y = \sin x°$ but compressed by a factor of 3 (the *period* is 120°).

The calculator gives $\sin^{-1}(0.5) = 30$, so the principal solution is given by

$$3x = 30 \Rightarrow x = 10.$$

The vertical lines on the graph below are at multiples of 60°. So you can see from the graph that the other solutions are 50°, 130°, 170°, 250° and 290°.



$y = \sin 3x$

Method 2: The principal value of $3x$ is $\sin^{-1}(0.5) = 30°$.

Therefore $3x = 30$ or $180 - 30 = 150$,

or $360 + 30$ or $360 + 150$

or $720 + 30$ or $720 + 150$

$\Rightarrow 3x = 30, 150, 390, 510, 750, 870$

$\Rightarrow x = 10, 50, 130, 170, 250, 290.$

Notice that with Method 2 you have to look at values of $3x$ in the range 0 to 1080 ($= 3 \times 360$), which is somewhat non-intuitive.

Exercise 2.1

1 Solve the following equations for $0 \le x < 360$. Give your answers to the nearest 0.1°.

(a) $\sin x° = 0.9$ (b) $\cos x° = 0.6$ (c) $\tan x° = 2$

(d) $\sin x° = -0.4$ (e) $\cos x° = -0.5$ (f) $\tan x° = -3$

2 Solve the following equations for $-180 \le x < 180$. Give your answers to the nearest 0.1°.

(a) $\sin x° = 0.9$ (b) $\cos x° = 0.6$ (c) $\tan x° = 2$

(d) $\sin x° = -0.4$ (e) $\cos x° = -0.5$ (f) $\tan x° = -3$

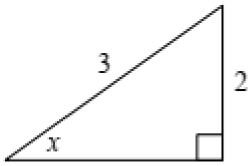
3 Solve the following equations for $0 \le x < 360$. Give your answers to the nearest 0.1°.

(a) $\sin 2x° = 0.829$ (b) $\cos 3x° = 0.454$ (c) $\tan 4x = 2.05$

(d) $\sin \frac{1}{2}x° = 0.8$ (e) $\cos \frac{1}{2}x° = 0.3$ (f) $\tan \frac{1}{3}x° = 0.7$

2.2 Other Trigonometric Methods

Suppose that you are told that $\sin x°$ is exactly $\frac{2}{3}$. Assuming that x is between 0° and 90°, you can find the exact values of $\cos x°$ and $\tan x°$ by drawing a right-angled triangle in which the opposite side and the hypotenuse are 2 and 3 respectively:



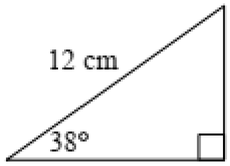
Now Pythagoras's Theorem tells you that the third, adjacent, side is $\sqrt{3^2 - 2^2} = \sqrt{5}$.

Hence $\cos x° = \dfrac{\sqrt{5}}{3}$ and $\tan x° = \dfrac{2}{\sqrt{5}}$.

This is preferable to using a calculator as the calculator does not always give exact values for this type of calculation. (Calculators can *in general* not handle irrational numbers exactly, although many are programmed to do so in simple cases.)

A further skill is being able to write down the lengths of the opposite and adjacent sides quickly when you know the hypotenuse. Some students like to do this using the sine rule, but it is not advisable to rely on the sine rule, especially in the mechanics section of A Level mathematics.

Example 1 Find the lengths of the opposite and adjacent sides in this triangle.



Solution Call the opposite and adjacent sides y and x respectively. Then

$\sin 38° = \dfrac{y}{12}$ so $y = 12 \sin 38° = 7.39$ cm (3 sf).

$\cos 38° = \dfrac{x}{12}$ so $x = 12 \cos 38° = 9.46$ cm (3 sf).

It should become almost automatic that the *opposite* side is (hypotenuse) × sin (angle)

and that the *adjacent* side is (hypotenuse) × cos (angle).

If you always have to work these out slowly you will find your progress, in mechanics in particular, is hindered.

Exercise 2.2

Do not use a calculator in this exercise.

1 In this question θ is in the range $0 \leq \theta < 90$.

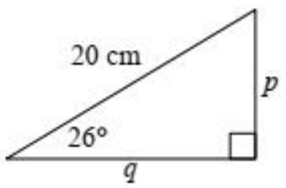
(a) Given that $\sin\theta = \frac{12}{13}$, find the exact values of $\cos\theta$ and $\tan\theta$.

(b) Given that $\tan\theta = \frac{6}{7}$, find the exact values of $\sin\theta$ and $\cos\theta$.

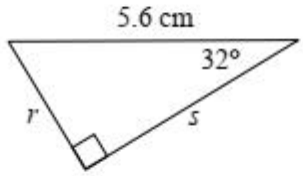
(c) Given that $\cos\theta = \frac{5}{8}$, find the exact values of $\sin\theta$ and $\tan\theta$.

2 Find expressions, of the form $a\sin\theta$ or $b\cos\theta$, for the sides labelled with letters in these triangles.

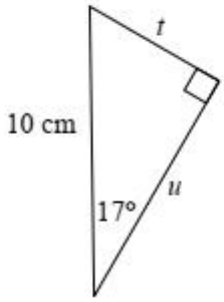
(a)



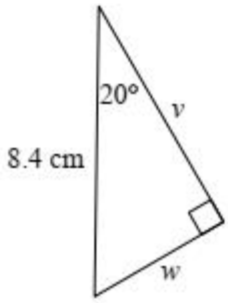
(b)



(c)



(d)



3 Graphs

No doubt you will have *plotted* many graphs of functions such as $y = x^2 - 3x + 4$ by working out the coordinates of points and plotting them on graph paper. But it is actually much more useful for A Level mathematics (and beyond) to be able to *sketch* the graph of a function. It might sound less challenging to be asked to draw a rough sketch than to plot an accurate graph, but in fact the opposite is true. The point is that in order to draw a quick sketch you have to understand the basic shape and some simple features of the graph, whereas to plot a graph you need very little understanding. Many professional mathematicians do much of their basic thinking in terms of shapes of graphs, and you will be more in control of your work, and understand it better, if you can do this too.

When you sketch a graph you are *not* looking for exact coordinates or scales. You are simply conveying the essential features:

- the basic shape
- where the graph hits the axes
- what happens towards the edges of your graph

The actual *scale* of the graph is irrelevant. For instance, it doesn't matter what the y-coordinates are.

3.1 Straight line graphs

I am sure that you are very familiar with the equation of a straight line in the form $y = mx + c$, and you have probably practised converting to and from the forms

$$ax + by + k = 0 \qquad \text{or} \qquad ax + by = k,$$

usually with a, b and k are integers. You need to be fluent in moving from one form to the other. The first step is usually to get rid of fractions by multiplying both sides by a common denominator.

Example 1 Write $y = \frac{3}{5}x - 2$ in the form $ax + by + k = 0$, where a, b and k are integers.

Solution Multiply both sides by 5: $5y = 3x - 10$

Subtract $5y$ from both sides: $0 = 3x - 5y - 10$

or $3x - 5y - 10 = 0$

In the first line it is a very common mistake to forget to multiply the 2 by 5.

It is a bit easier to get everything on the right instead of on the left of the equals sign, and this reduces the risk of making sign errors.

In plotting or sketching lines whose equations are written in the form $ax + by = k$, it is useful to use the *cover-up rule*:

Example 2 Draw the graph of $3x + 4y = 24$.

Solution Put your finger over the "$3x$". You see "$4y = 24$".

This means that the line hits the y-axis at $(0, 6)$.

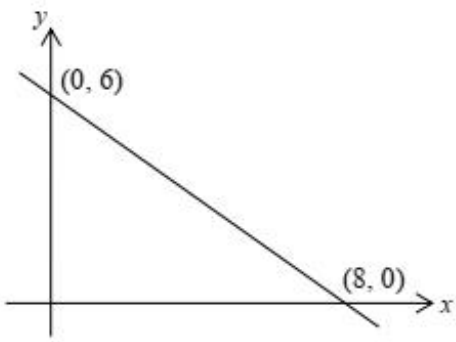
Repeat for the "$4y$". You see "$3x = 24$".

This means that the line hits the x-axis at $(8, 0)$.

[NB: *not* the point $(8, 6)$!]

Mark these points in on the axes.

You can now draw the graph.



Exercise 3.1

1 Rearrange the following in the form $ax + by + c = 0$ or $ax + by = c$ as convenient, where a, b and c are integers and $a > 0$.

(a) $y = 3x - 2$ (b) $y = \frac{1}{2}x + 3$

(c) $y = -\frac{3}{4}x + 3$ (d) $y = \frac{7}{2}x - \frac{5}{4}$

(e) $y = -\frac{2}{3}x + \frac{3}{4}$ (f) $y = \frac{4}{7}x - \frac{2}{3}$

2 Rearrange the following in the form $y = mx + c$. Hence find the gradient and the y-intercept of each line.

(a) $2x + y = 8$ (b) $4x - y + 9 = 0$

(c) $x + 5y = 10$ (d) $x - 3y = 15$

(e) $2x + 3y + 12 = 0$ (f) $5x - 2y = 20$

(g) $3x + 5y = 17$ (h) $7x - 4y + 18 = 0$

3 Sketch the following lines. Show on your sketches the coordinates of the intercepts of each line with the x-axis and with the y-axis.

(a) $2x + y = 8$ (b) $x + 5y = 10$

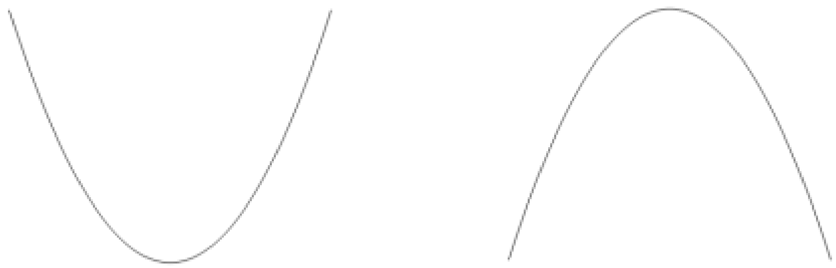
(c) $2x + 3y = 12$ (d) $3x + 5y = 30$

(e) $3x - 2y = 12$ (f) $4x + 5y + 20 = 0$

3.2 Basic shapes of curved graphs

You need to know the names of standard types of expressions, and the graphs associated with them.

(a) The graph of a **quadratic** function (e.g. $y = 2x^2 + 3x + 4$) is a ***parabola:***

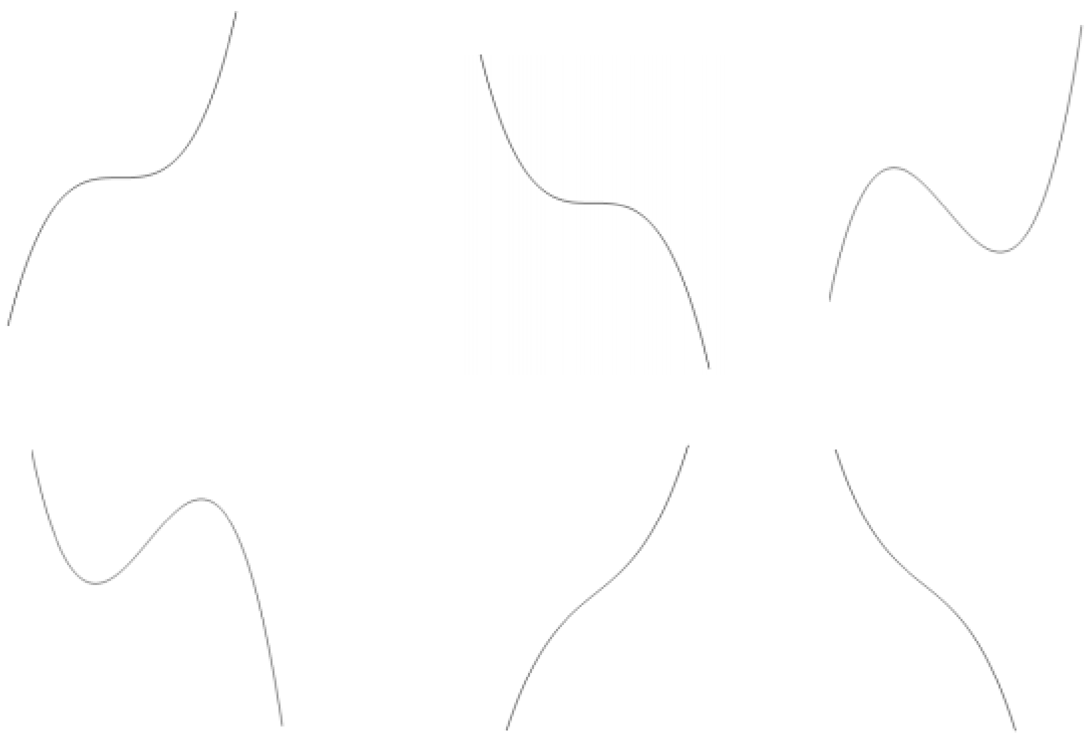


Notes:

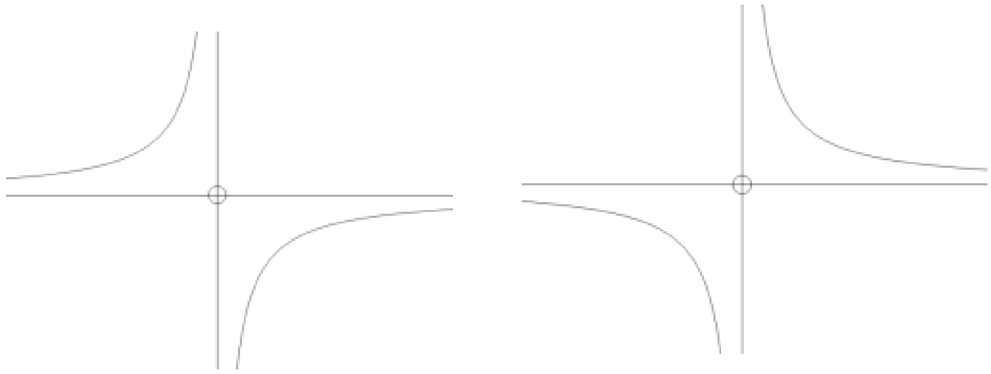
- Parabolas are symmetric about a vertical line.
- They are not U-shaped, so the sides never reach the vertical. Neither do they dip outwards at the ends.

These are wrong:

(b) The graph of a **cubic** function (e.g. $y = 2x^3 - 3x^2 + 4x - 5$) has no particular name; it's usually referred to simply as a **cubic graph**. It can take several possible shapes:

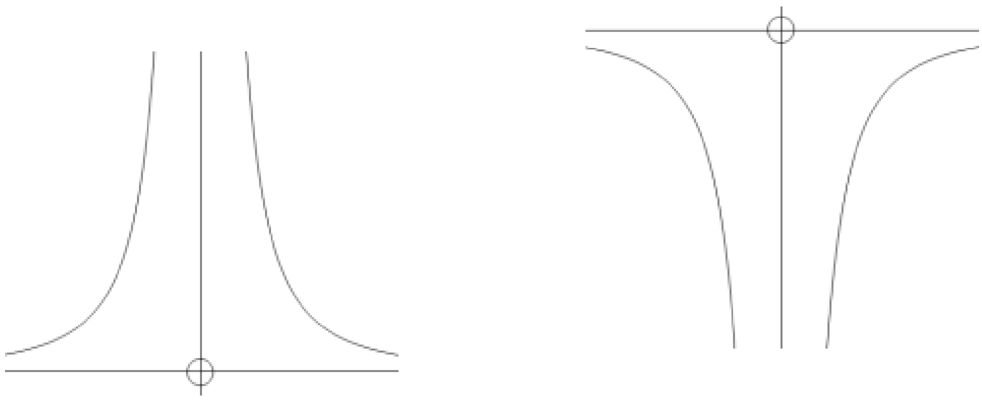


(c) The graph of $y = \frac{\text{a number}}{x}$ is a ***hyperbola***:



The graph of a hyperbola gets closer and closer to the axes without ever actually touching them. This is called **asymptotic** behaviour, and the axes are referred to as the **asymptotes** of this graph.

(d) The graph of $y = \frac{\text{a number}}{x^2}$ is similar (but not identical) to a hyperbola to the right but is in a different quadrant to the left:

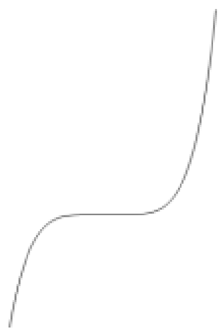


(e) Graphs of higher *even* powers

$y = x^4$ ($y = x^6$ etc. are similar):

(f) Graphs of higher *odd* powers

$y = x^5$ ($y = x^7$ etc. are similar):



Which way up? This is determined by the *sign of the highest power*.

If the sign is positive, the *right-hand* side is (eventually) *above the x-axis*.

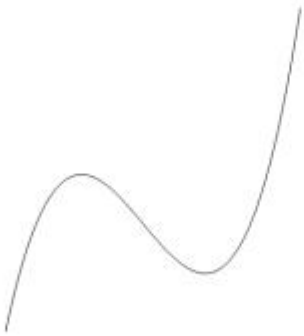
This is because for big values of x the highest power dominates the expression.

(If $x = 1000$, x^3 is bigger than $50x^2$).

Examples $y = x^2 - 3x - 1$ $y = 10 - x^2$

These are often referred to (informally!) as ***happy*** and ***sad*** parabolas respectively ☺ ☹ .

$y = x^3 - 3x - 2$ $y = 2 - x - x^5$



Exercise 3.2

Sketch (do not *plot*) the general shape of the graphs of the following curves.

Axes are not required but can be included in the questions marked with an asterix.

1 $y = x^2 - 3x + 2$

2 $y = -x^2 + 5x + 1$

3 $y = 1 - x^2$

4 $y = (x - 2)(x + 4)$

5 $y = (3 - x)(2 + x)$

6 $y = (1 - x)(5 - x)$

7 $y = x^3$

8 $y = -x^3$

9* $y = \frac{3}{x}$

10* $y = -\frac{2}{x}$

11 $y = (x - 2)(x - 3)(x + 1)$

12* $y = \frac{2}{x^2}$

13 Sketch on the same axes the general shape of the graphs of $y = x^2$ and $y = x^4$.

14 Sketch on the same axes the general shape of the graphs of $y = x^3$ and $y = x^5$.

3.3 Factors

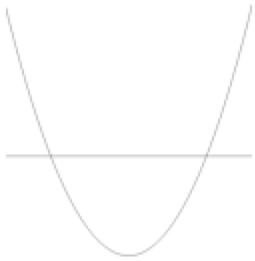
Factors are crucial when curve-sketching.

They tell you where the curve meets the x-axis.

Do not multiply out brackets!

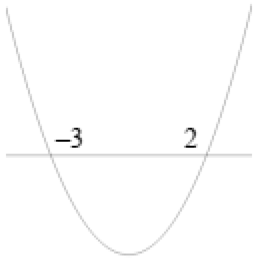
Example Sketch the graph of $y = (x - 2)(x + 3)$.

Solution The graph is a *positive* (happy!) *parabola* so start by drawing the *correct shape* with a *horizontal axis* across it.

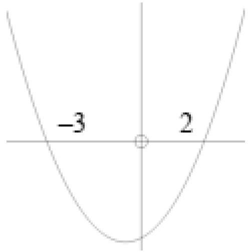


The factors tell you that it hits the x-axis at $x = -3$ and $x = 2$.

Mark these on your sketch:

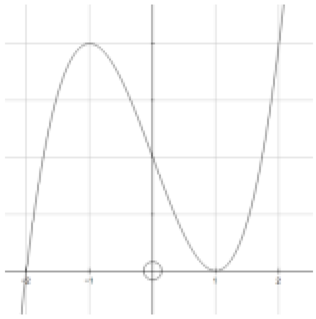


and only now put in the y-axis, which is clearly slightly nearer 2 than –3:



Note: the lowest point on the graph is *not* on the y-axis. (Because the graph is symmetric, it is at $x = -\frac{1}{2}$.)

Repeated factors



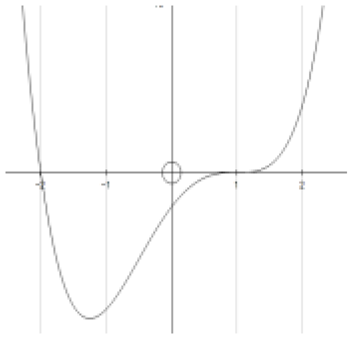
Suppose you want to sketch $y = (x-1)^2(x+2)$.

You know there is an intercept at $x = -2$.

At $x = 1$ the graph *touches* the axes, as if it were

the graph of $y = (x-1)^2$ there.

[More precisely, it is very like $y = 3(x-1)^2$ there. That is because, close to $x = 1$, the $(x-1)^2$ factor changes rapidly, while $(x+2)$ remains close to 3.]



Likewise, the graph of $y = (x+2)(x-1)^3$

looks like $y = (x-1)^3$ close to $x = 1$.

[Again, more precisely, it is

very like $y = 3(x-1)^3$ there.]

Exercise 3.3

Sketch the curves in questions 1–21. Use a different diagram for each.
Show the x-coordinates of the intersections with the x-axis.

1 $y = x^2$

2 $y = (x - 1)(x - 3)$

3 $y = (x + 2)(x - 4)$

4 $y = x(x - 3)$

5 $y = (x + 2)(3x - 2)$

6 $y = x(4x + 3)$

7 $y = -x(x - 3)$

8 $y = (2 - x)(x + 1)$

9 $y = (3 - x)(2 + x)$

10 $y = (x + 2)(x - 1)(x - 4)$

11 $y = x(x - 1)(x + 2)$

12 $y = -x(x - 1)(x + 2)$

13 $y = (3 - x)(2 - x)(1 - x)$

14 $y = (x - 1)^2(x - 3)$

15 $y = (x - 1)(x - 3)^2$

16 $y = (x + 1)^3$

17 $y = (2 - x)(x + 1)^3$

18 $y = (x + 1)(x + 2)(x - 1)(x - 2)$

19 $y = -(x + 3)(x + 2)(x - 1)(x - 4)$

20 $y = (x - 2)^2(x + 2)^2$

21 $y = (x - 1)(x - 2)^2(x - 3)^3$

22 (a) Sketch the graph of $y = x^2$.

(b) Sketch $y = 2x^2$ on the same axes.

(c) Sketch $y = x^2 + 1$ on the same axes.

23 (a) Sketch the graph of $y = \sqrt{x}$.

(b) Sketch $y = 2\sqrt{x}$ on the same axes.

24 (a) Sketch the graph of $y = \frac{1}{x}$.

(b) Sketch $y = \frac{1}{x} + 1$ on the same axes.

25 (a) Sketch the graph of $y = \frac{1}{x^2}$.

(b) Sketch $y = \frac{2}{x^2}$ on the same axes.

26 (a) Sketch the graph of $y = x^3$.

(b) Sketch $y = 2x^3$ on the same axes.

27 (a) Sketch the graph of $y = x^4$.

(b) Sketch $y = 3x^4$ on the same axes.

28 (a) Sketch the graph of $y = x^3 - 4x$.

[Hint: It cuts the x-axis at -2, 0 and 2.]

(b) Sketch $y = 2x^3 - 8x$ on the same axes.

29 (a) Sketch the graph of $y = x^4 - x^2$.

[Hint: It cuts the x-axis at 1 and -1, and touches the axis at 0.]

(b) Sketch $y = -x^4 + x^2$ on the same axes.

30 Sketch, on separate axes, the following graphs. Show the x-coordinates of the intersections with the x-axis.

(a) $y = 4 - x^2$

(b) $y = (x - 2)(x + 1)$

(c) $y = -(x - 2)(x + 1)$

(d) $y = x(x + 4)$

(e) $y = (x - 2)^2$

(f) $y = -(x + 1)^2$

(g) $y = (1 - x)(2 + x)$

Answers, hints and comments

Exercise 2.1

1 (a) 64.2, 115.8 (b) 53.1, 306.9 (c) 63.4, 243.4

(d) 203.6, 336.4 (e) 120, 240 (f) 108.4, 288.4

2 (a) 64.2, 115.8 (b) 53.1, –53.1 (c) 63.4, –116.6

(d) –23.6, –156.4 (e) 120, –120 (f) –71.5, 108.4

3 (a) 28, 62, 208, 242 (b) 21, 99, 141, 219, 261, 339

(c) 16, 61, 106, 151, 196, 241, 286, 331 (d) 106.2, 253.7

(e) 145.1 (f) 105

Exercise 2.2

1 (a) $\frac{5}{13}, \frac{12}{5}$ (b) $\frac{6}{\sqrt{85}}, \frac{7}{\sqrt{85}}$ (c) $\frac{\sqrt{39}}{8}, \frac{\sqrt{39}}{5}$

2 (a) $p = 20 \sin 26°$, $q = 20 \cos 26°$ (b) $r = 5.6 \sin 32°$, $s = 5.6 \cos 32°$

(c) $t = 10 \sin 17°$, $u = 10 \cos 17°$ (d) $v = 8.4 \cos 20°$, $w = 8.4 \sin 20°$

Exercise 3.1

1 (a) $3x - y = 2$ (b) $x - 2y + 6 = 0$

(c) $3x + 4y = 12$ (d) $14x - 4y = 5$

(e) $8x + 12y = 9$ (f) $12x - 21y = 14$

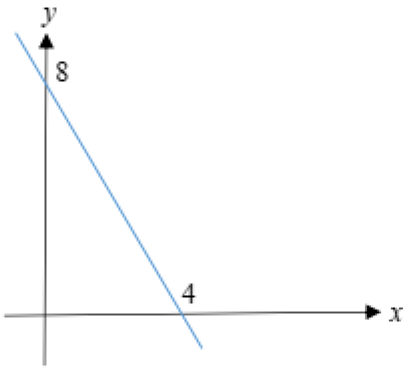
2 (a) $y = -2x + 8$; -2, 8 (b) $y = 4x + 9$; 4, 9

(c) $y = -\frac{1}{5}x + 2$; $-\frac{1}{5}$, 2 (d) $y = \frac{1}{3}x - 5$; $\frac{1}{3}$, -5

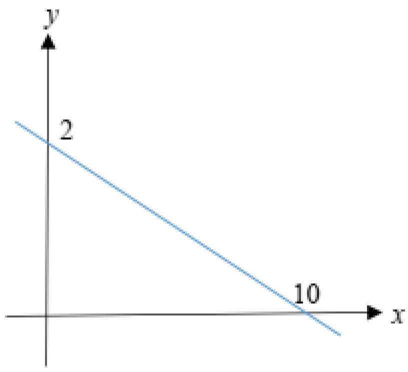
(e) $y = -\frac{2}{3}x - 4$; $-\frac{2}{3}$, -4 (f) $y = \frac{5}{2}x - 10$; $\frac{5}{2}$, -10

(g) $y = -\frac{3}{5}x + \frac{17}{5}$; $-\frac{3}{5}$, $\frac{17}{5}$ (h) $y = \frac{7}{4}x + \frac{9}{2}$; $\frac{7}{4}$, $\frac{9}{2}$

3 (a)



(b)



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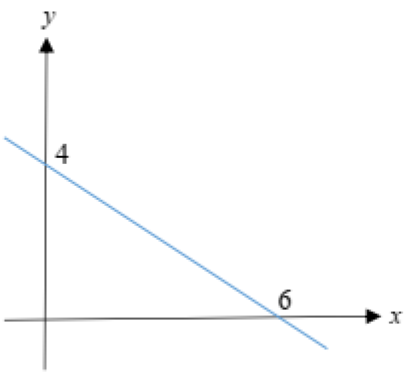

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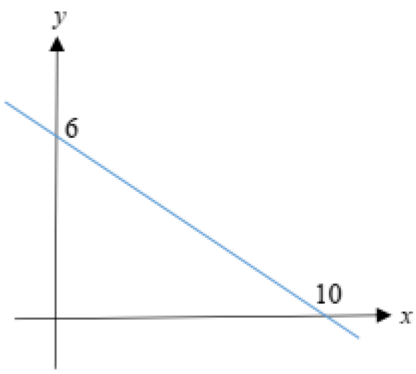

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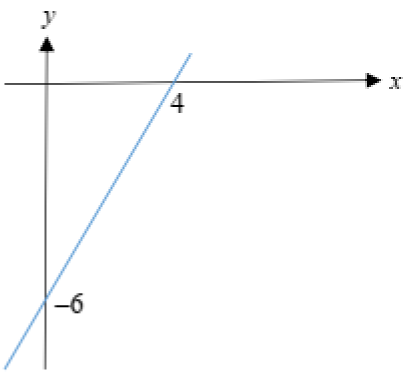

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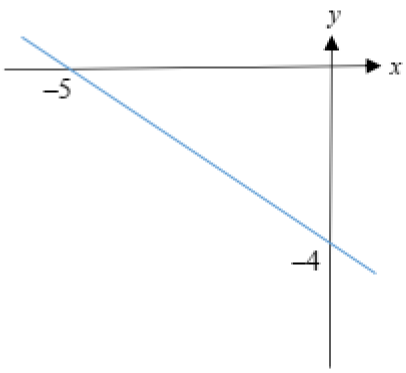

Exercise 3.2

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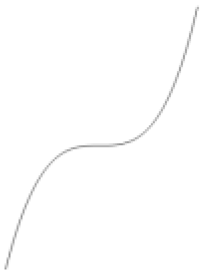

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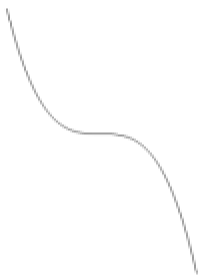

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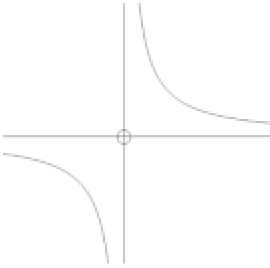

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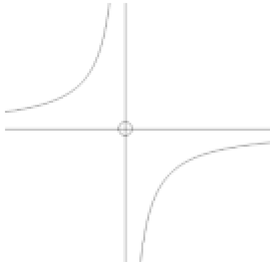

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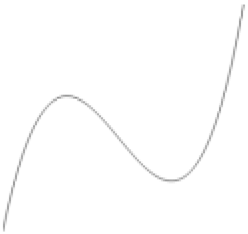

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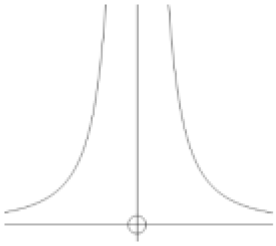

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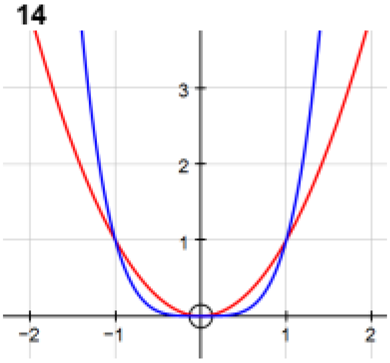
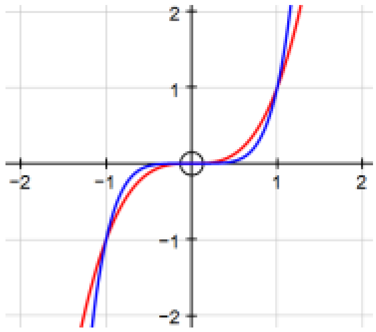


red: $y = x^2$ blue: $y = x^4$



red: $y = x^3$ blue: $y = x^5$

Exercise 3.3

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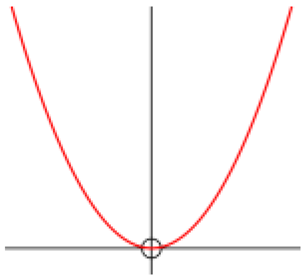

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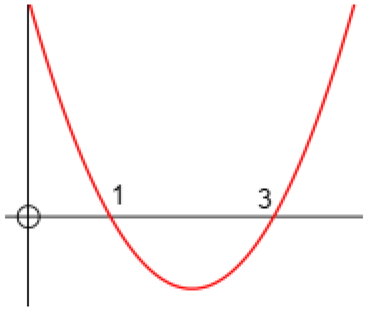

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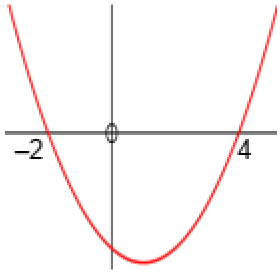

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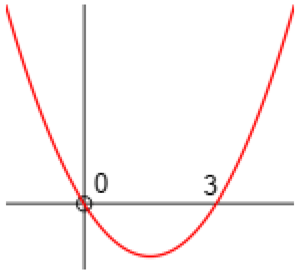

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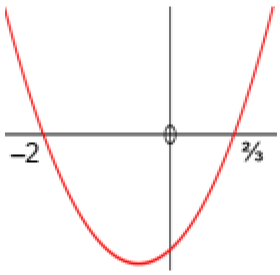

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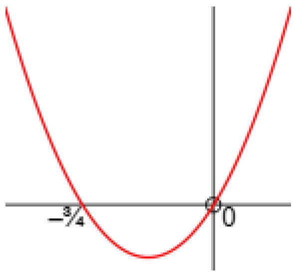

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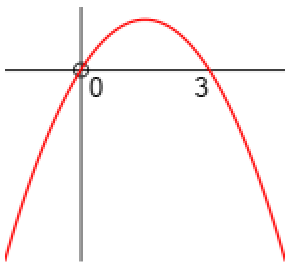

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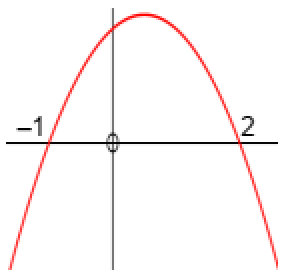

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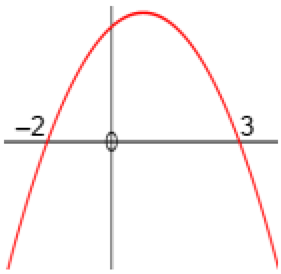

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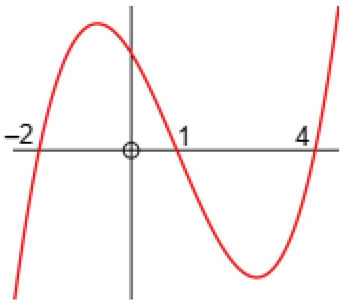

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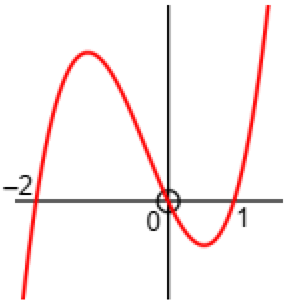

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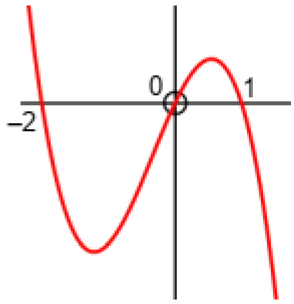

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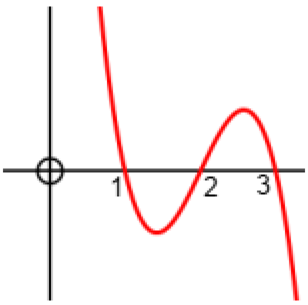

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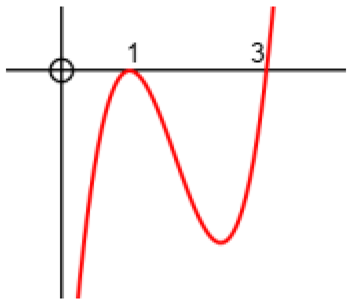

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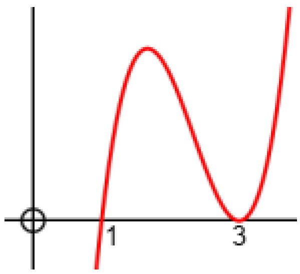

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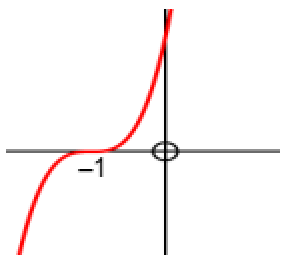

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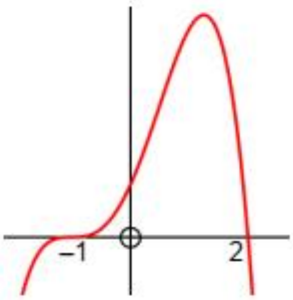

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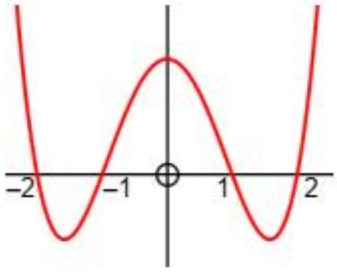

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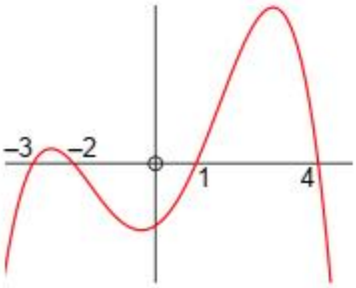

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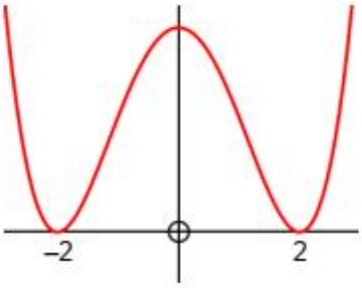

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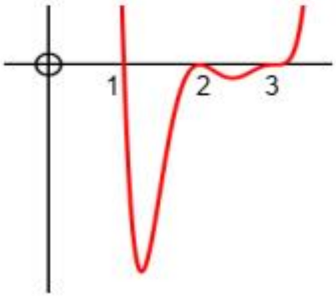
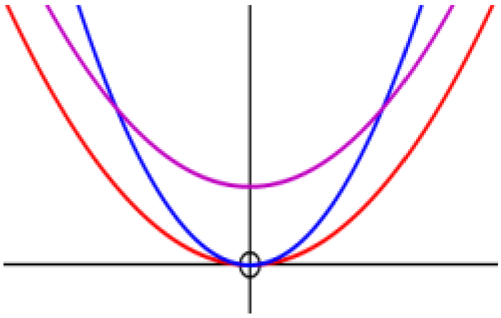


[In this graph in particular, do NOT worry about the y-coordinates of the minimum points.]

22

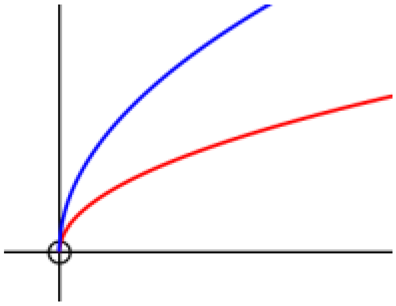


red: $y = x^2$

blue: $y = 2x^2$

purple: $y = x^2 + 1$

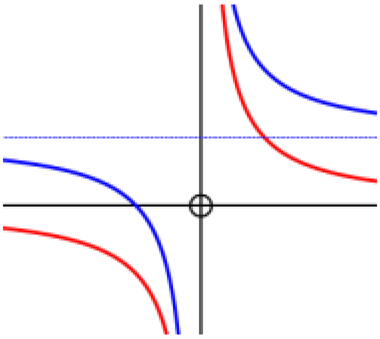
23



red: $y = \sqrt{x}$

blue: $y = 2\sqrt{x}$

24

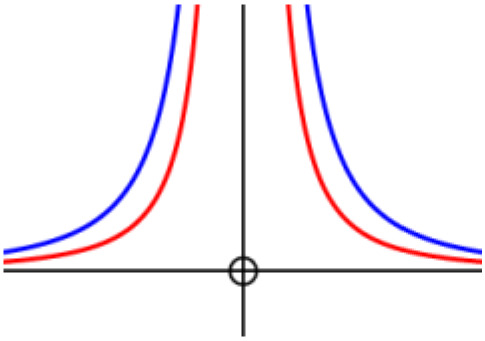


red: $y = \frac{1}{x}$

blue: $y = \frac{1}{x} + 1$

blue dotted: $y = 1$ *[horizontal asymptote]*

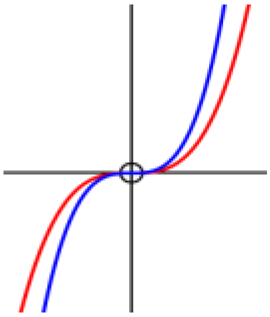
25



red: $y = \frac{1}{x^2}$

blue: $y = \frac{2}{x^2}$

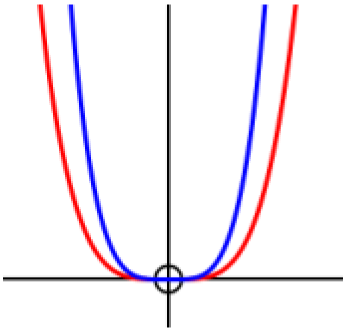
26



red: $y = x^3$

blue: $y = 2x^3$

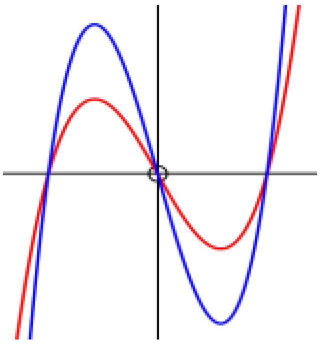
27



red: $y = x^4$

blue: $y = 3x^4$

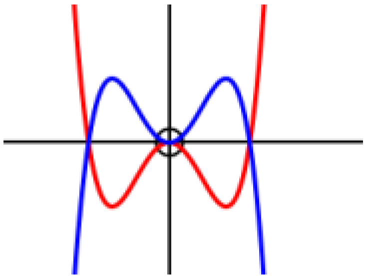
28



red: $y = x^3 - 4x$

blue: $y = 2x^3 - 8x$

29

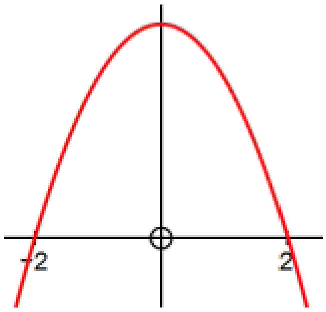


red: $y = x^4 - x^2$

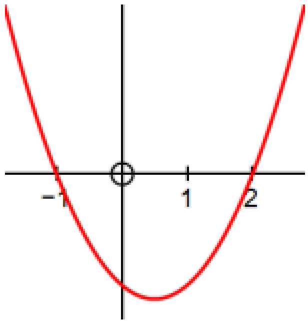
blue: $y = -x^4 + x^2$

30

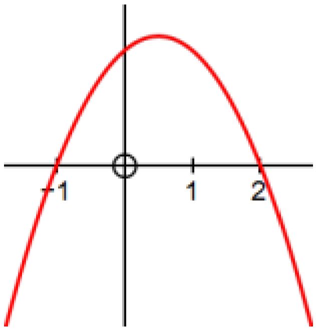
(a)



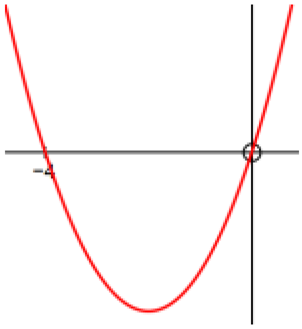
(b)



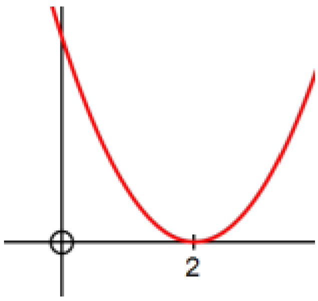
(c)



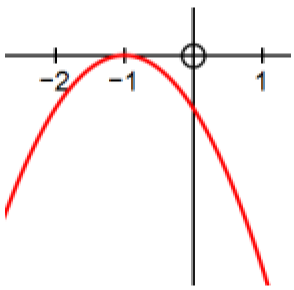
(d)

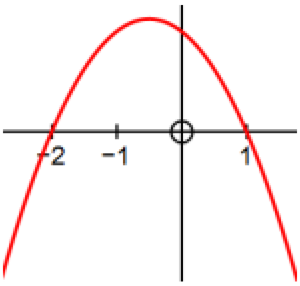


(e)



(f)



(g) 

NOTE: in parts (b), (c) and (g) in particular, the maximum or minimum point is *not* on the y-axis.