Mathematics

Y12 to Y13 Mathematics Summer Independent Learning

June to August 2024

There are two tasks to work through.

2 Tasks

Please read the following instructions very carefully and ensure you label and collate all your work ready for checking in September.

For your first Maths lesson please bring

- A large A4 folder with dividers.
- These instructions with the tables filled in (print out/copy the tables onto A4 paper).
- *A list of questions you need to ask prior to doing your initial test.*

Task 1: Consolidation of Pure Mathematics Y12 content

1. Complete all questions, ideally in retrieval conditions.
2. Note down any topics struggled with, and use the below link to Jack Brown videos to help improve your understanding. https://sites.google.com/site/tlmaths314/home/a-level-maths-2017/full-a-level
3. Review any topics of particular concern using your gapped notes

Question / topic	Video(s) *(Tick)*	Gapped notes booklet used/ main issues?

Task 2 – Topic based consolidation of recent 'Y13 content'

1. Complete all questions for the 3 topics.
2. Mark and correct all questions using answers provided.
3. Update review sheet with details of work completed.

Topic	**Any points of weakness you need to address**
Reciprocal Trigonometry	
Differentiation exam questions	
Forces	

Task 1: Consolidation of Pure Mathematics Y12 content

Question 1

Fully justify your workings

Write each of the following surd expressions as simple as possible.

a) $2\sqrt{32}+\sqrt{18}-3\sqrt{8}$.

b) $\dfrac{22}{4-\sqrt{5}}$.

Question 2

$$f(x)=3x^2+12x+8,\ x\in\mathbb{R}.$$

a) Express $f(x)$ in the form $a(x+b)^2+c$, where a, b and c are integers.

b) State the minimum value of $f(x)$.

c) Solve the equation $f(x)=0$, giving the answers as exact simplified surds.

Question 3

A cubic graph is defined by

$$f(x)\equiv x^3-3x^2-4x+12,\ x\in\mathbb{R}.$$

a) Show that $(x-3)$ is a factor of $f(x)$.

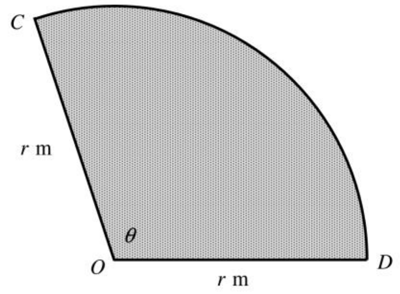
b) Hence factorize $f(x)$ as the product of three linear factors.

c) Sketch the graph of $f(x)$.

The sketch must include the coordinates of any points where the graph of $f(x)$ meets the coordinate axes.

d) State the roots of the equation $f(x-2)$

Question 4



A circular sector OCD, subtending an angle θ radians at its centre O, has a radius of r m.

The sector has an area of 0.25 m^2 and a perimeter of 2 m.

Determine the values of r and θ.

Question 5

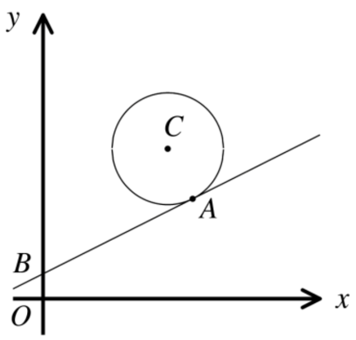
Solve each of the following equations, giving the final answers correct to three significant figures, where appropriate.

a) $6^{3x+2} = 30$.

b) $\log_4(12y+5) - \log_4(1-y) = 2$.

c) $8^{2t} - 8^t - 6 = 0$.

Question 6



The figure above shows a circle with centre at C with equation

$$x^2 + y^2 - 10x - 12y + 56 = 0.$$

The tangent to the circle at the point $A(6,4)$ meets the y axis at the point B.

a) Find an equation of the tangent to the circle at A.

b) Determine the area of the triangle ABC.

Question 7

Solve the following trigonometric equation in the range given.

$$4\tan^2\theta\cos\theta = 15,\ 0 \le \theta < 360°.$$

Question 8

$$f(x) = \sqrt{27x^3 + 1},\ x \ge -\tfrac{1}{3}.$$

The graph of $f(x)$ is stretched horizontally by scale factor 3, to produce the graph of $g(x)$.

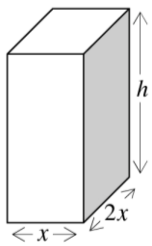
Determine in its simplest form the equation of $g(x)$.

Question 9

$$y = 3 - \cos 2x°,\ 0 \le x \le 360.$$

Describe geometrically the three transformations that map the graph of $y = \cos x°$ onto the graph of $y = 3 - \cos 2x°$.

Question 10



The figure above shows the design of a fruit juice carton with capacity of $1000\ \text{cm}^3$.

The design of the carton is that of a closed cuboid whose base measures x cm by $2x$ cm, and its height is h cm.

a) Show that the surface area of the carton, $A\ \text{cm}^2$, is given by

$$A = 4x^2 + \frac{3000}{x}.$$

b) Find the value of x for which A is stationary.

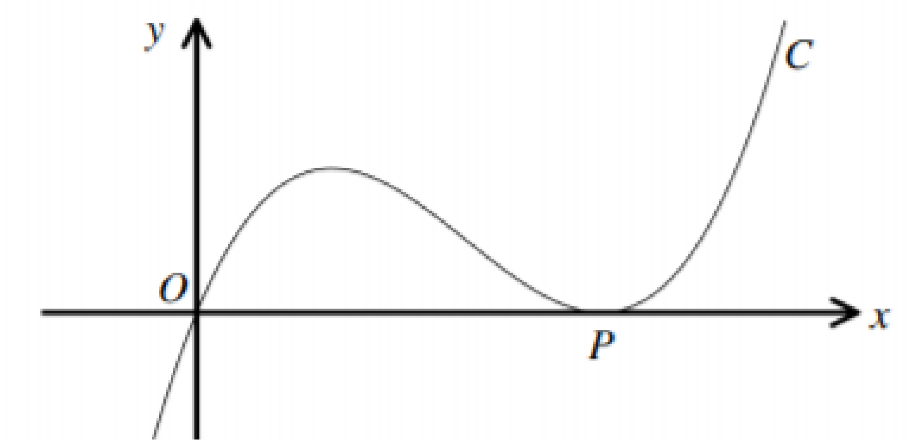
c) Calculate the minimum value for A, justifying fully the fact that it is indeed the minimum value of A.

Question 11

Use differentiation from first principles, to show that for $f(x) = x^4, x \in R$

$$f'(x) = 4x^3$$

Question 12



The figure above shows the cubic curve C which meets the coordinates axes at the origin O and at the point P.

The gradient function of C is given by

$$f'(x) = 3x^2 - 8x + 4.$$

a) Find an equation for C.

b) Determine the coordinates of P.

c) Calculate the area bounded by curve and the x-axis.

Question 13

Remember you can use column vectors or i,j,k notation!

Relative to a fixed origin O, the points A, B and C have respective position vectors $2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$, $5\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$ and $7\mathbf{j} - 4\mathbf{k}$.

a) Given that $ABCD$ is a parallelogram, determine the position vector of D.

b) Determine the distance AC and hence calculate the angle BAC

Question 14

The sum of the first 20 terms of an arithmetic series is 1070.

The sum of its fifth term and its tenth term is 65.

a) Find the first term and the common difference of the series.

b) Calculate the sum of the first 30 terms of the series.

Question 15

The second term of a geometric series is 4 and its sum to infinity is 18.

a) Show that the common ratio r of the series is a solution of the equation

$$9r^2 - 9r + 2 = 0.$$

b) Find the two possible values of r and the corresponding values of the first term of the series.

The sum of the first n terms of the series is denoted by S_n.

c) Given that r takes the larger of the two values found in part (b) determine the smallest value of n for which S_n exceeds 17.975.

Question 16

$$y = \sqrt{4 - 12x}, \quad -\frac{1}{3} < x < \frac{1}{3}.$$

a) Find the binomial expansion of y in ascending powers of x up and including the term in x^3, writing all coefficients in their simplest form.

b) Hence find the coefficient of x^2 in the expansion of

$$(12x - 4)(4 - 12x)^{\frac{1}{2}}.$$

Question 17

Given that

$$\frac{2x^3 + x^2 - 4x + 1}{x^2 + x - 2} \equiv Ax + B + \frac{C}{x + D},$$

use polynomial division, or another appropriate method, to find the value of each of the constants A, B, C and D.

[Solutions begin on next page]

Task 1 solutions

Question 1

(a)
$$2\sqrt{32}+\sqrt{18}-3\sqrt{8}$$
$$= 2\sqrt{16\times 2}+\sqrt{9\times 2}-3\sqrt{4\times 2}$$
$$= 2\times 4\sqrt{2}+3\sqrt{2}-3\times 2\sqrt{2}$$
$$= 8\sqrt{2}+3\sqrt{2}-6\sqrt{2}$$
$$= 5\sqrt{2}$$

(b)
$$\frac{22}{4-\sqrt{5}} = \frac{22(4+\sqrt{5})}{(4-\sqrt{5})(4+\sqrt{5})}$$
$$= \frac{22(4+\sqrt{5})}{16+\cancel{4\sqrt{5}}-\cancel{4\sqrt{5}}-5}$$
$$= \frac{22(4+\sqrt{5})}{11}$$
$$= 2(4+\sqrt{5})$$
$$= 8+2\sqrt{5}$$

Question 2

(a)
$$f(x) = 3x^2+12x+8$$
$$\Rightarrow f(x) = 3\left[x^2+4x+\frac{8}{3}\right]$$
$$\Rightarrow f(x) = 3\left[(x+2)^2-4+\frac{8}{3}\right]$$
$$\Rightarrow f(x) = 3(x+2)^2-12+8$$
$$\Rightarrow f(x) = 3(x+2)^2-4$$

(b)
$$f(x)_{MIN} = -4$$
(OCCURS WHEN $x=-2$

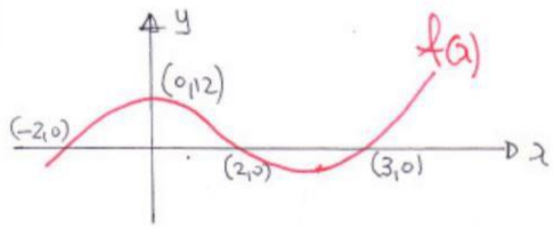
(c)
$$f(x)=0$$
$$\Rightarrow 3x^2+12x+8=0$$
$$\Rightarrow 3(x+2)^2-4=0$$
$$\Rightarrow 3(x+2)^2=4$$
$$\Rightarrow (x+2)^2=\frac{4}{3}$$
$$\Rightarrow x+2=\pm\sqrt{\frac{4}{3}}$$
$$\Rightarrow x+2=\pm\frac{2}{\sqrt{3}}$$
$$\Rightarrow x+2=\pm\frac{2}{3}\sqrt{3}$$
$$\Rightarrow x=-2\pm\frac{2}{3}\sqrt{3}$$

Question 3

(a)
$$f(x)=x^3-3x^2-4x+12$$
$$f(3)=27-27-12+12=0$$
$\therefore (x-3)$ IS A FACTOR

(b) BY LONG DIVISION OR GROUPING
$$f(x)=x^3-3x^2-4x+12$$
$$f(x)=x^2(x-3)-4(x-3)$$
$$f(x)=(x-3)(x^2-4)$$
$$f(x)=(x-3)(x-2)(x+2)$$

(c)

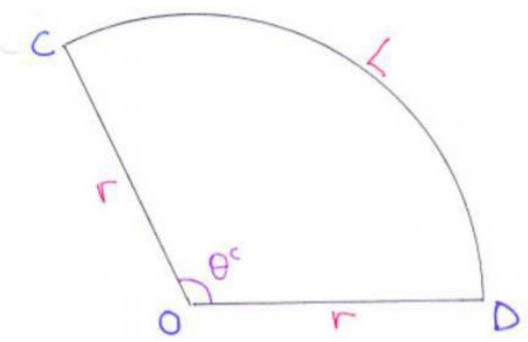


$+x^3 \Rightarrow$

$x=0 \Rightarrow y=12$

$y=0 \Rightarrow x = -2,\ 2,\ 3$

Question 4



- $A = 0.25$

$\frac{1}{2}r^2\theta^c = \frac{1}{4}$

$r^2\theta = \frac{1}{2}$

$\boxed{2r^2\theta = 1}$

- $P = 2$

$2r + L = 2$

$2r + r\theta = 2$

$\boxed{r\theta = 2 - 2r}$

$\Rightarrow 2r(r\theta) = 1$

$2r(2-2r) = 1$

$4r - 4r^2 = 1$

$0 = 4r^2 - 4r + 1$

$0 = (2r-1)^2$

$r = \frac{1}{2} = 0.5m$ //

USING $r\theta = 2 - 2r$

$\frac{1}{2}\theta = 1$

$\theta = 2^c$ //

Question 5

a) $6^{3x+2} = 30$

$\log_6 30 = 3x + 2$

$x = \dfrac{(\log_6 30) - 2}{3}$

$x = -0.0339$

b) $\log_4(12y+5) - \log_4(1-y) = 2$

$\Rightarrow \log_4\left(\frac{12y+5}{1-y}\right) = 2$

$\Rightarrow \log_4\left(\frac{12y+5}{1-y}\right) = 2\log_4 4$

$\Rightarrow \log_4\left(\frac{12y+5}{1-y}\right) = \log_4 16$

$\Rightarrow \frac{12y+5}{1-y} = 16$

$\Rightarrow 12y + 5 = 16 - 16y$

$\Rightarrow 28y = 11$

$\Rightarrow y = \frac{11}{28}$ //

$\simeq 0.393$

(c) $8^{2t} - 8^t - 6 = 0$

$\Rightarrow (8^t)^2 - (8^t) - 6 = 0$

LET $a = 8^t$

$\Rightarrow a^2 - a - 6 = 0$

$\Rightarrow (a-3)(a+2) = 0$

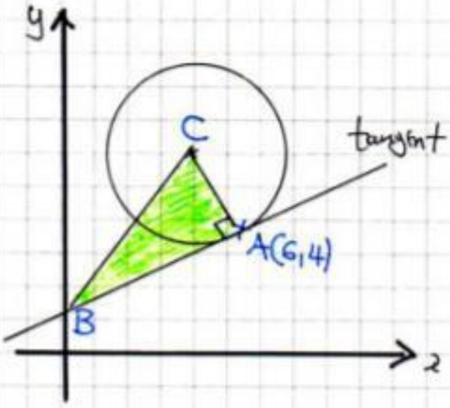
$\Rightarrow a = 3, -2$

$8^t = 3$ or -2

$8^t = 3$	$8^t \neq -2$
$\log_8 3 = t$	$8^t > 0$
	for all t
$t = 0.528$	

Question 6

a) REWRITING THE EQUATION OF THE CIRCLE, TO READ THE CENTRE



$$x^2+y^2-10x-12y+56=0$$
$$x^2-10x+y^2-12y+56=0$$
$$(x-5)^2-25+(y-6)^2-36+56=0$$
$$(x-5)^2+(y-6)^2=5$$

$\therefore C(5,6)$ & $r=\sqrt{5}$

FIND THE GRADIENT OF AC, WHERE C(5,6) & A(6,4)

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{4-6}{6-5}=\frac{-2}{1}=-2$$

USING PERPENDICULAR GRADIENT OF $+\frac{1}{2}$ WE "OBTAIN THE TANGENT"

$$y-y_0=m(x-x_0)$$
$$y-4=\frac{1}{2}(x-6)$$
$$2y-8=x-6$$
$$2y=x+2 \quad \text{or} \quad y=\frac{1}{2}x+1$$

b) FIND THE CO-ORDINATES OF B

WHEN $x=0$ $\quad 2y=2$

$y=1$

$\therefore B(0,1)$

I would say just draw a RA triangle, reduces the likelihood of mistakes with negatives!

FIND THE DISTANCE AB, WHERE A(6,4) & B(0,1)

$$\Rightarrow d=\sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$$
$$\Rightarrow |AB|=\sqrt{(1-4)^2+(0-6)^2}$$
$$\Rightarrow |AB|=\sqrt{9+36}$$
$$\Rightarrow |AB|=\sqrt{45}=3\sqrt{5}$$

HENCE THE REQUIRED AREA IS GIVEN BY

$$\Rightarrow \text{AREA}=\frac{1}{2}|AB||AC|$$
$$\Rightarrow \text{AREA}=\frac{1}{2}\times 3\sqrt{5}\times\sqrt{5}$$
$$\Rightarrow \text{AREA}=\frac{15}{2}$$

Question 7

$$4\tan^2\theta\cos\theta = 15$$

$$\Rightarrow 4\left(\frac{\sin\theta}{\cos\theta}\right)^2\cos\theta = 15$$

$$\Rightarrow \frac{4\sin^2\theta}{\cos^2\theta} \times \cos\theta = 15$$

$$\Rightarrow \frac{4\sin^2\theta}{\cos\theta} = 15$$

$$\Rightarrow 4\sin^2\theta = 15\cos\theta$$

$$\Rightarrow 4(1-\cos^2\theta) = 15\cos\theta$$

$$\Rightarrow 4 - 4\cos^2\theta = 15\cos\theta$$

$$\Rightarrow 0 = 4\cos^2\theta + 15\cos\theta - 4$$

$$\Rightarrow (4\cos\theta - 1)(\cos\theta + 4) = 0$$

$$\Rightarrow \cos\theta = \frac{1}{4}, \ \cancel{-4}$$

$$\arccos\left(\tfrac{1}{4}\right) = 75.5°$$

$$\theta = 75.5° \pm 360n$$
$$\theta = 284.5 \pm 360n \qquad n = 0,1,2,3..$$

$$\theta_1 = 75.5°$$
$$\theta_2 = 284.5°$$

Question 8

$$f(x) \longmapsto f\left(\tfrac{1}{3}x\right)$$

$$\sqrt{27x^3+1} \qquad \sqrt{27\left(\tfrac{1}{3}x\right)^3+1}$$

$$\sqrt{x^3+1}$$

Question 9

(a) $\cos(x) \longmapsto \cos(2x) \longmapsto -\cos 2x \longmapsto -\cos 2x + 3$

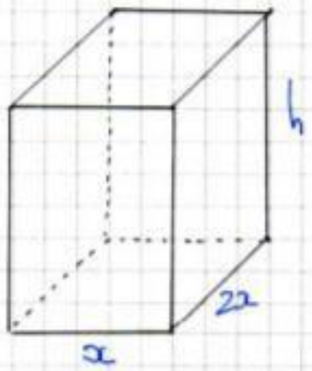
STRETCH, HORIZONTALLY SCALE FACTOR $\frac{1}{2}$

REFLECTION IN THE x AXIS

TRANSLATION, UPWARDS BY 3 UNITS

Question 10

a)



CONSTRAINT $V = 1000 \text{ cm}^3$

$\Longrightarrow V = x(2x)h$

$\Longrightarrow 1000 = 2x^2h$

$\Longrightarrow x^2h = 500$

$xh = \frac{500}{x}$

$\boxed{6xh = \frac{3000}{x}}$

SURFACE AREA (A cm^2)

$\Longrightarrow A = 2\left[2x^2 + xh + 2xh\right]$

$\Longrightarrow A = 4x^2 + 6xh$

$\Longrightarrow A = 4x^2 + \frac{3000}{x}$ //

AS REQUIRED

b) $A = 4x^2 + 3000x^{-1}$

$\Longrightarrow \frac{dA}{dx} = 8x - 3000x^{-2}$

FOR STATIONARY VALUES $\frac{dA}{dx} = 0$

$\Longrightarrow 8x - \frac{3000}{x^2} = 0$

$\Longrightarrow 8x = \frac{3000}{x^2}$

$\Longrightarrow 8x^3 = 3000$

$\Longrightarrow x^3 = 375$

$\Longrightarrow x = \sqrt[3]{375} \approx 7.21 \text{ cm}$ //

c) $A = 4x^2 + \frac{3000}{x}$

$\Rightarrow A_{MIN} = 4(7.21...)^2 + \frac{3000}{7.21...}$

$\Rightarrow A_{MIN} \approx 624 \text{ cm}^2$

TO JUSTIFY IT IS A MIN, USE 2^{ND} DERIVATIVE

$\Rightarrow \frac{dA}{dx} = 8x - 3000x^{-2}$

$\Rightarrow \frac{d^2A}{dx^2} = 8 + 6000x^{-3}$

$\Rightarrow \frac{d^2A}{dx^2} = 8 + \frac{6000}{x^3}$

$\Rightarrow \left.\frac{d^2A}{dx^2}\right|_{x=7.21...} = 8 + \frac{6000}{(7.21...)^3} = 24 > 0$

INDEED A MINIMUM

Question 11

THE DERIVATIVE IS FORMALLY GIVEN BY

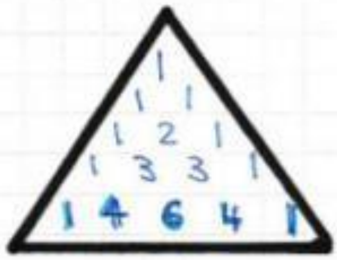
$$f'(x) = \lim_{h \to 0} \left[\frac{f(x+h) - f(x)}{h} \right]$$

IN THIS CASE WE HAVE

$$f(x) = x^4$$
$$f(x+h) = (x+h)^4$$

EXPANDING BINOMIALLY WE HAVE

$$(x+h)^4 = 1x^4h^0 + 4x^3h^1 + 6x^2h^2 + 4x^1h^3 + 1x^0h^4$$
$$(x+h)^4 = x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4$$



TIDYING UP NEXT

$$f(x+h) - f(x) = (x+h)^4 - x^4 = (\cancel{x^4} + 4x^3h + 6x^2h^2 + 4xh^3 + h^4) - \cancel{x^4}$$

FINALLY WE HAVE

$$f'(x) = \lim_{h \to 0} \left[\frac{f(x+h) - f(x)}{h} \right] = \lim_{h \to 0} \left[\frac{4x^3h + 6x^2h^2 + 4xh^3 + h^4}{h} \right]$$

$$= \lim_{h \to 0} \left[4x^3 + \cancel{6x^2h} + \cancel{4xh^2} + \cancel{h^3} \right]$$

$$= \underline{4x^3}$$

Question 12

(a) IF $f'(x) = 3x^2 - 8x + 4$

THEN

$$f(x) = \int 3x^2 - 8x + 4 \, dx$$

$$f(x) = x^3 - 4x^2 + 4x + c$$

BUT CUBIC GOES THROUGH (0,0)

$$0 = 0 - 0 + 0 + c$$

$$\therefore c = 0$$

$$\therefore f(x) = x^3 - 4x^2 + 4x$$

(b) $f(x) = x^3 - 4x^2 + 4x$

$$f(x) = x(x^2 - 4x + 4)$$

$$f(x) = x(x-2)^2$$

$\therefore$ when $y = 0$

$$x = 0, \ 2$$

$\therefore P(2,0)$

(c)

$$\int_0^2 x^3 - 4x^2 + 4x \, dx$$

$$\left[\frac{x^4}{4} - \frac{4x^3}{3} + 2x^2 \right]_0^2$$

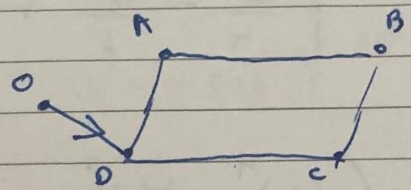
$$\left(\frac{(2)^4}{4} - \frac{4(2)^3}{3} + 2(2)^2 \right) - (0)$$

$$\frac{4}{3}$$

Question 13

(a) $\overrightarrow{OA} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$ $\overrightarrow{OB} = \begin{pmatrix} 5 \\ -3 \\ 4 \end{pmatrix}$ $\overrightarrow{OC} = \begin{pmatrix} 0 \\ 7 \\ -4 \end{pmatrix}$

Parallelogram diagram



$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD}$
(other options too)

$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{BC}$ parallelogram so $\overrightarrow{AD} = \overrightarrow{BC}$

$\overrightarrow{OD} = \overrightarrow{OA} + \underbrace{\overrightarrow{BO}}_{[-\overrightarrow{OB}]} + \overrightarrow{OC}$

$\overrightarrow{OD} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + \begin{pmatrix} -5 \\ 3 \\ -4 \end{pmatrix} + \begin{pmatrix} 0 \\ 7 \\ -4 \end{pmatrix}$ $\boxed{\overrightarrow{OD} = \begin{pmatrix} -3 \\ 13 \\ -9 \end{pmatrix}}$

1b) Distance AC, need vector $\overrightarrow{AC}$

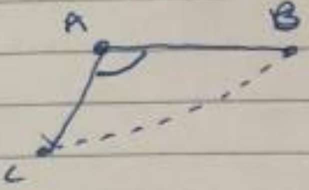
$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC}$

$\overrightarrow{AC} = \begin{pmatrix} -2 \\ -3 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 7 \\ -4 \end{pmatrix}$

$\overrightarrow{AC} = \begin{pmatrix} -2 \\ 4 \\ -3 \end{pmatrix}$ $\therefore$ $|\overrightarrow{AC}|$ or $AC = \sqrt{2^2 + 4^2 + 3^2}$

$|\overrightarrow{AC}|$ or $AC = \sqrt{29}$

Angle BAC will
required cosine rule
with 3 sides



$|\vec{AC}| = \sqrt{29}$

$\vec{BC} = \begin{pmatrix} -5 \\ 10 \\ -8 \end{pmatrix}$ Used workings from (a)

$\therefore |\vec{BC}| = \sqrt{5^2 + 10^2 + 8^2}$

$|\vec{BC}| = \sqrt{189}$

lastly $\vec{AB} = \vec{AO} + \vec{OB}$

$\vec{AB} = \begin{pmatrix} -2 \\ -3 \\ 1 \end{pmatrix} + \begin{pmatrix} 5 \\ -3 \\ 4 \end{pmatrix}$ $\vec{AB} = \begin{pmatrix} 3 \\ -6 \\ 5 \end{pmatrix}$

$\therefore |\vec{AB}| = \sqrt{3^2 + 6^2 + 5^2}$

$|\vec{AB}| = \sqrt{70}$

Now

$BC^2 = AC^2 + AB^2 - 2 \times AC \times AB \cos A$

$189 = 29 + 70 - 2 \times \sqrt{29} \times \sqrt{70} \cos A$

$\cos A = \dfrac{70 + 29 - 189}{2\sqrt{29}\sqrt{70}}$

$A = 177.2^\circ$ (odd!)

Question 14

(a) $\boxed{S_n = \frac{n}{2}[2a + (n-1)d]}$

$\Rightarrow 1070 = \frac{20}{2}[2a + 19d]$

$\Rightarrow 1070 = 10[2a + 19d]$

$\Rightarrow \boxed{107 = 2a + 19d}$

$u_5 + u_{10} = 65$

$(a+4d) + (a+9d) = 65$

$\boxed{2a + 13d = 65}$

$2a = 107 - 19d$

$2a = 65 - 13d$

$107 - 19d = 65 - 13d$

$42 = 6d$

$d = 7$

& $2a = 65 - 13d$

$2a = 65 - 13 \times 7$

$2a = 65 - 91$

$2a = -26$

$a = -13$

(b) $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_{30} = \frac{30}{2}[2 \times (-13) + 29 \times 7]$

$\Rightarrow S_{30} = 15[-26 + 203]$

$\Rightarrow S_{30} = 15 \times 177$

$\Rightarrow S_{30} = 2655$

$$\begin{array}{r} 1770 \\ 885 \\ \hline 2655 \end{array}$$

Question 15

$u_2 = 4$

$S_\infty = 18$

(a) • $u_n = ar^{n-1}$

$\boxed{4 = ar}$

• $S_\infty = \frac{a}{1-r}$

$18 = \frac{a}{1-r}$

$\boxed{18 - 18r = a}$

$4 = (18 - 18r)r$

$4 = 18r - 18r^2$

$18r^2 - 18r + 4 = 0$

$9r^2 - 9r + 2 = 0$ As required

(b) $(3r-1)(3r-2) = 0$

$r = \frac{1}{3}$ or $\frac{2}{3}$

$\boxed{\text{Now } a = \frac{4}{r}}$

$\therefore a = \frac{4}{1/3} = 12$ or $\frac{4}{2/3} = 6$

$\therefore a = 12$ with $r = \frac{1}{3}$

OR

$a = 6$ with $r = \frac{2}{3}$

(c)

$r = \frac{2}{3}, \quad a = 6$ $\qquad S_n = \frac{a(1-r^n)}{1-r}$

$S_n > 17.975$ easier to solve = first

$17.975 = \frac{6\left(1-\left(\frac{2}{3}\right)^n\right)}{1-\frac{2}{3}}$ don't expand bracket...

$17.975 = 18\left(1-\left(\frac{2}{3}\right)^n\right)$

$0.9986\dot{1} = 1-\left(\frac{2}{3}\right)^n$

$\left(\frac{2}{3}\right)^n = 0.0013\dot{8}$

$\log_{\frac{2}{3}} 0.0013\dot{8} = n$

$n = 16.226$

$\underline{n = 17}$

question says exceeds, so $n = 17$.

check:

$S_{16} = \frac{6\left(1-\left(\frac{2}{3}\right)^{16}\right)}{1-\frac{2}{3}} = 17.9725$

$S_{17} = \frac{6\left(1-\left(\frac{2}{3}\right)^{17}\right)}{1-\frac{2}{3}} = 17.98$

Question 16

(a) $y = \sqrt{4-12x} = (4-12x)^{\frac{1}{2}} = 4^{\frac{1}{2}}(1-3x)^{\frac{1}{2}} = 2(1-3x)^{\frac{1}{2}}$

$= 2\left[1 + \frac{1}{2}(-3x) + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{1\times 2}(-3x)^2 + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{1\times 2\times 3}(-3x)^3 + O(x^4)\right]$

$= 2\left[1 - \frac{3}{2}x - \frac{9}{8}x^2 - \frac{27}{16}x^3 + O(x^4)\right]$

$= 2 - 3x - \frac{9}{4}x^2 - \frac{27}{8}x^3 + O(x^4)$

(b) $(12x-4)(4-12x)^{\frac{1}{2}}$

$(12x-4)\left(2 - 3x - \frac{9}{4}x^2 - \frac{27}{8}x^3 + O(x^4)\right)$

$-36x^2$ $\quad 9x^2$

$\therefore -27x^2$

$\therefore -27$

Question 17

$$\frac{2x^3 + x^2 - 4x + 1}{x^2 + x - 2}$$

	$2x$	-1	R
x^2	$2x^3$	$-x^2$	$+x$
$+x$	$+2x^2$	$-x$	-1
-2	$-4x$	$+2$	

lots of other methods are possible for polynomial division, all giving the same answer.

Also a quick check on my graphical calculator shows $(x-1)$ could be cancelled to begin....

So given $2x - 1 + \dfrac{x-1}{x^2+x-2}$

$$2x - 1 + \frac{(x-1)}{(x-1)(x+2)}$$

$$2x - 1 + \frac{1}{x+2}$$

$$\frac{2x^3 + x^2 - 4x + 1}{(x-1)(x+2)} = Ax + B + \frac{C}{x + D}$$

Even if you spot this factorise, unless you 'guess' what D is, a partial fraction style method is difficult!

Task 2 Solutions can be found with questions

Task 2 – Topic based consolidation of recent 'Y13 content'

Topic 1 - Reciprocal Trigonometry

Question 1

Solve, for $-\pi \le x \le \pi$, the equation,

$$5 \cos x + \cot x = 0$$

Give your answers to 2 decimal places where appropriate.

Question 2

(a) Use the identity $\cos^2\theta + \sin^2\theta = 1$ to prove that $\tan^2\theta = \sec^2\theta - 1$ **(2)**

(b) Solve, for $0 \le \theta \le 360$, the equation,

$$\tan^2\theta + \sec^2\theta + 5\sec\theta = 2$$

Give your answers to 1 decimal place. **(5)**

Question 3

(a) Use the identity $\cos^2\theta + \sin^2\theta = 1$ to prove that $\text{cosec}^2\theta = 1 + \cot^2\theta$ **(2)**

(b) Solve, for $0 \le \theta \le 2\pi$, the equation,

$$\text{cosec}^2\theta + \cot^2\theta = 3$$

Give your answers in terms of π. **(5)**

Question 4

(a) Show that the expression $\dfrac{1 - \sin x}{\cos x} + \dfrac{\cos x}{1 - \sin x}$ can be written as $2\sec x$.

[4 marks]

(b) Hence solve the equation

$$\frac{1 - \sin x}{\cos x} + \frac{\cos x}{1 - \sin x} = \tan^2 x - 2$$

giving the values of x to the nearest degree in the interval $0° \leqslant x < 360°$.

[4 marks]

Question 5

It is given that

$$\frac{\operatorname{cosec} x - \sin x}{\cot x \cos^2 x} \equiv \sec x .$$

a) Prove the validity of the above trigonometric identity.

b) Hence, or otherwise, solve the trigonometric equation

$$\tan^2 x - \sec x = \frac{\operatorname{cosec} x - \sin x}{2\cot x \cos^2 x}, \ 0 \le x < 360° .$$

[Solutions begin on next page]

Question 1

$$5\cos x + \cot x = 0$$

$$5\cos x + \frac{\cos x}{\sin x} = 0$$

$$5\cos x \sin x + \cos x = 0$$

$$\cos x\,(5\sin x + 1) = 0$$

$\cos x = 0 \qquad \sin x = -1/5$

$x = \frac{1}{2}\pi, -\frac{1}{2}\pi \qquad x = -0.20, -2.94$

Question 2

1a)

$$\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$\tan^2\theta = \sec^2\theta - 1$$

b/

$$\tan^2\theta + \sec^2\theta + 5\sec\theta = 2$$

$$\sec^2\theta - 1 + \sec^2\theta + 5\sec\theta = 2$$

$$2\sec^2\theta + 5\sec\theta - 3 = 0$$

$$(2\sec\theta - 1)(\sec\theta + 3) = 0$$

$\sec\theta = \frac{1}{2} \qquad \sec\theta = -3$

$\cos\theta \neq 2 \qquad \cos\theta = -\frac{1}{3}$

$\times \qquad \theta = 109.5, 250.5$

$-1 \leq \cos x \leq 1$

Question 3

2a/

$$\frac{\cos^2\theta}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$\cot^2\theta + 1 = \operatorname{cosec}^2\theta$$

$$\operatorname{cosec}^2\theta = 1 + \cot^2\theta$$

b/

$$\operatorname{cosec}^2\theta + \cot^2\theta = 3$$

$$1 + \cot^2\theta + \cot^2\theta = 3$$

$$2\cot^2\theta = 2$$

$$\cot^2\theta = 1$$

$$\tan^2\theta = 1$$

$$\tan\theta = \pm 1$$

$\tan\theta = 1$ $\qquad$ $\tan\theta = -1$

$\theta = \frac{\pi}{4}, \frac{5}{4}\pi$ $\qquad$ $\theta = -\frac{1}{4}\pi, \frac{3}{4}\pi, \frac{7}{4}\pi$

$$\underline{\underline{\frac{\pi}{4}}}, \underline{\underline{\frac{3}{4}\pi}}, \underline{\underline{\frac{5}{4}\pi}}, \underline{\underline{\frac{7}{4}\pi}}$$

Question 4

Q	Solution	Mark	Total	Comment
8(a)	$\frac{1-\sin x}{\cos x}+\frac{\cos x}{1-\sin x}=\frac{(1-\sin x)^2+\cos^2 x}{\cos x(1-\sin x)}$	M1		Combining fractions correctly
	$=\frac{1-2\sin x+\sin^2 x+\cos^2 x}{\cos x(1-\sin x)}$			
	$=\frac{1-2\sin x+1}{\cos x(1-\sin x)}$	m1		Using $\sin^2 x+\cos^2 x=1$
	$=\frac{2-2\sin x}{\cos x(1-\sin x)}$ *or* $\frac{2(1-\sin x)}{\cos x(1-\sin x)}$	A1		Must have factorised denominator
	$=\frac{2}{\cos x}$			
	$=2\sec x$	A1	4	AG, both expressions seen

(b)				
	$\tan^2 x - 2 = 2\sec x$			
	$\sec^2 x - 1 - 2 = 2\sec x$			Using $\tan^2 x = \sec^2 x - 1$, OE
	$\sec^2 x - 2\sec x - 3\ (=0)$	B1		Or $3\cos^2 x + 2\cos x - 1\ (=0)$
	$(\sec x - 3)(\sec x + 1)\quad (=0)$			Correctly factorising their expression or substituting into formula
	$\sec x = 3 \quad or \quad -1$	A1		Or $\cos x = \frac{1}{3} \quad or \quad -1$
	$\sec x = 3 \quad \Rightarrow \quad x = 71°\ ,\quad 289°$	B1		no extras inside the interval
	$\sec x = -1 \quad \Rightarrow \quad x = 180°$	B1		$0 \le x < 360°$,−1 EE
			4	

Question 5

(a) $LHS = \dfrac{\operatorname{cosec}x - \sin x}{\cot x \cos^2 x} = \dfrac{\frac{1}{\sin x} - \sin x}{\frac{\cos x}{\sin x} \times \cos^2 x}$

MULTIPLY TOP BOTTOM BY $\sin x$

$= \dfrac{1 - \sin^2 x}{\cos^3 x} = \dfrac{\cos^2 x}{\cos^3 x} = \dfrac{1}{\cos x} = \sec x = RHS$

(b) $2(\tan^2 x - \sec x) = \dfrac{\operatorname{cosec}x - \sin x}{\cot x \cos^2 x}$

$\Rightarrow 2\tan^2 x - 2\sec x = \sec x$

$\Rightarrow 2(\sec^2 x - 1) - 2\sec x = \sec x$

$\Rightarrow 2\sec^2 x - 3\sec x - 2 = 0$

$\Rightarrow (2\sec x + 1)(\sec x - 2) = 0$

$\Rightarrow \sec x = -\frac{1}{2},\ 2$

$\Rightarrow \cos x = \cancel{-2},\ \frac{1}{2}$

$-1 \le cosx \le 1$

$x_1 = 60$

$x_2 = 300$

Question 1

The curve C has equation

$$y = \frac{x^2}{2x+1}, \ x \neq -\frac{1}{2}$$

a) Show clearly that

$$\frac{dy}{dx} = \frac{2x^2+2x}{(2x+1)^2}.$$

b) Find the coordinates of the stationary points of C.

[*the nature of these stationary points need not be determined*]

Question 2

A curve C has equation

$$y = \sqrt{x-3}, \ x > 3.$$

Find an equation of the normal to C at the point where $x = 7$

Question 3

The curve C has equation

$$y = x \ln x, \ x > 0.$$

Find the exact coordinates of the turning point of C.

Question 4

A curve has equation

$$y = \left(x^2 + 3x + 2\right)\cos 2x .$$

Determine an equation of the tangent to the curve at the point where the curve crosses the y axis.

Question 5

A curve C has equation

$$y = x\mathrm{e}^{2x}, \ x \in \mathbb{R} .$$

Show that an equation of the tangent to C at the point where $x = \frac{1}{2}$ is

$$2y = \mathrm{e}(4x - 1).$$

[Solutions begin on next page]

Question 1

(a) $y = \dfrac{x^2}{2x+1}$

$\dfrac{dy}{dx} = \dfrac{(2x+1)(2x) - x^2(2)}{(2x+1)^2} = \dfrac{4x^2+2x-2x^2}{(2x+1)^2} = \dfrac{2x^2+2x}{(2x+1)^2}$ //

(b) MIN/MAX

$\dfrac{dy}{dx} = 0 \qquad \dfrac{2x^2+2x}{(2x+1)^2} = 0$

$2x^2+2x=0$

$2x(x+1)=0$

$x = 0, -1 \qquad y = 0, \dfrac{1}{-1} = -1 \qquad \therefore (0,0)$ & $(-1,-1)$ //

Question 2

$y = \sqrt{x-3} = (x-3)^{\frac{1}{2}}$

$\dfrac{dy}{dx} = \frac{1}{2}(x-3)^{-\frac{1}{2}}$

$\left.\dfrac{dy}{dx}\right|_{x=7} = \dfrac{1}{4}$

when $x=7$ $y=2$

NORMAL GRADIENT $= -4$ $\quad (7,2)$

$y - y_0 = m(x - x_0)$

$y - 2 = -4(x-7)$

$y - 2 = -4x + 28$

$y + 4x = 30$ //

Question 3

$y = x\ln x$

$\dfrac{dy}{dx} = 1 \times \ln x + x \times \dfrac{1}{x}$

$\dfrac{dy}{dx} = \ln x + 1$

FOR MIN/MAX $\dfrac{dy}{dx} = 0$

$\ln x + 1 = 0$

$\ln x = -1$

$x = e^{-1}$

$x = \dfrac{1}{e}$

AND

$y = x\ln x$

$y = \dfrac{1}{e}\ln\dfrac{1}{e}$

$y = -\dfrac{1}{e}$

$\therefore (e^{-1}, -e^{-1}) = \left(\dfrac{1}{e}, -\dfrac{1}{e}\right)$

Question 4

$$y = (x^2+3x+2)\cos 2x$$

$$\frac{dy}{dx} = (2x+3)\cos 2x + (x^2+3x+2)(-2\sin 2x)$$

$$\left.\frac{dy}{dx}\right|_{x=0} = 3\cos 0 - \cancel{4\sin 0}$$

$$\left.\frac{dy}{dx}\right|_{x=0} = 3$$

Also when $x=0$

$$y = 2\cos 0 = 2$$

$\therefore m=3 \quad (0,2)$

$$\therefore y = 3x+2$$

Question 5

$$y = xe^{2x}$$

$$\frac{dy}{dx} = 1\times e^{2x} + x\times(2e^{2x})$$

$$\frac{dy}{dx} = e^{2x} + 2xe^{2x}$$

$$\frac{dy}{dx} = e^{2x}(1+2x)$$

when $x=\frac{1}{2}$, $y=\frac{1}{2}e$, $\left.\frac{dy}{dx}\right|_{x=\frac{1}{2}} = 2e$

$\therefore$ TANGENT: $y-y_0 = m(x-x_0)$

$$\Rightarrow y-\tfrac{1}{2}e = 2e(x-\tfrac{1}{2})$$

$$\Rightarrow y-\tfrac{1}{2}e = 2ex-e$$

$$\Rightarrow 2y-e = 4ex-e$$

$$\Rightarrow 2y = 4ex-e$$

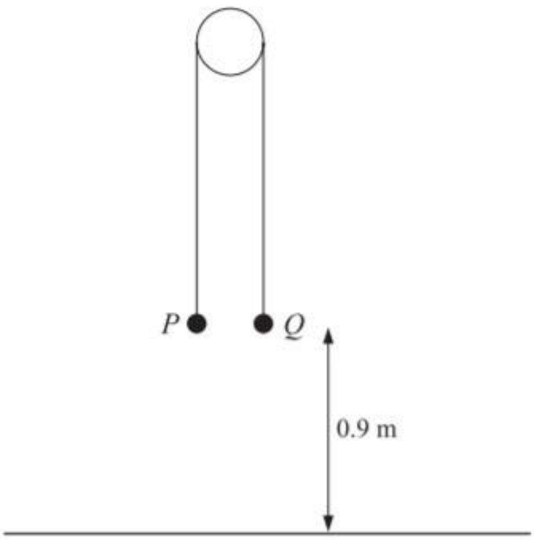
$$\Rightarrow 2y = e(4x-1)$$

AS REQUIRED

Topic 3 - Forces

Forces - Non – slopes

Question 1



Particles P and Q, of masses 0.6 kg and 0.2 kg respectively, are attached to the ends of a light inextensible string which passes over a smooth fixed peg. The particles are held at rest with the string taut. Both particles are at a height of 0.9 m above the ground (see diagram). The system is released and each of the particles moves vertically. Find

(i) the acceleration of P and the tension in the string before P reaches the ground, [5]

(ii) the time taken for P to reach the ground. [2]

Question 2

A car is towing a trailer along a straight horizontal road by means of a horizontal tow-rope. The mass of the car is 1400 kg. The mass of the trailer is 700 kg. The car and the trailer are modelled as particles and the tow-rope as a light inextensible string. The resistances to motion of the car and the trailer are assumed to be constant and of magnitude 630 N and 280 N respectively. The driving force on the car, due to its engine, is 2380 N. Find

(a) the acceleration of the car,

(3)

(b) the tension in the tow-rope.

(3)

When the car and trailer are moving at 12 m s^{-1}, the tow-rope breaks. Assuming that the driving force on the car and the resistances to motion are unchanged,

(c) find the distance moved by the car in the first 4 s after the tow-rope breaks.

(6)

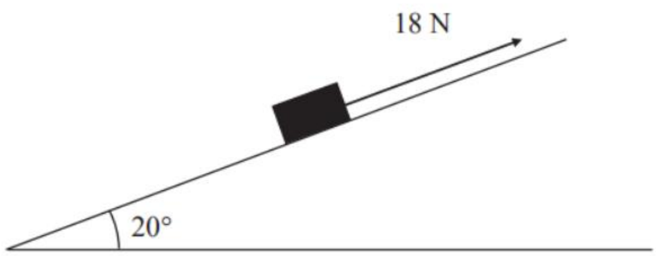
(d) State how you have used the modelling assumption that the tow-rope is inextensible.

(1)

Forces - Slopes

Question 1

Figure 2



A box of mass 2 kg is pulled up a rough plane face by means of a light rope. The plane is inclined at an angle of 20° to the horizontal, as shown in Figure 2. The rope is parallel to a line of greatest slope of the plane. The tension in the rope is 18 N. The coefficient of friction between the box and the plane is 0.6. By modelling the box as a particle, find

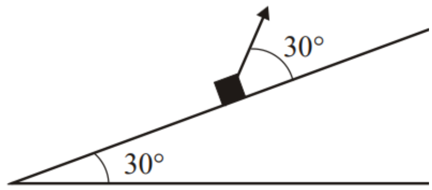
(*a*) the normal reaction of the plane on the box,

(3)

(*b*) the acceleration of the box.

(5)

Question 2



A small parcel of mass 2 kg moves on a rough plane inclined at an angle of 30° to the horizontal. The parcel is pulled up a line of greatest slope of the plane by means of a light rope which is attached to it. The rope makes an angle of 30° with the plane, as shown in the diagram above. The coefficient of friction between the parcel and the plane is 0.4.

Given that the tension in the rope is 24 N,

(a) find, to 2 significant figures, the acceleration of the parcel.

(8)

Extension

The rope now breaks. The parcel slows down and comes to rest.

(b) Show that, when the parcel comes to this position of rest, it immediately starts to move down the plane again.

(4)

(c) Find, to 2 significant figures, the acceleration of the parcel as it moves down the plane after it has come to this position of instantaneous rest.

(3)

[Solutions begin on next page]

Forces; Non – slopes

Question 1

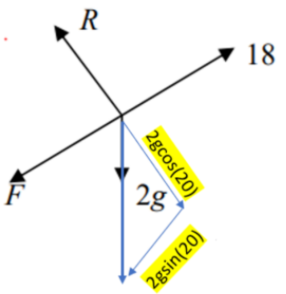
4 (i)		M1		For applying Newton's second law to P or to Q (3 terms)
	$0.6\,g - T = 0.6a$	A1		
	$T - 0.2\,g = 0.2a$	A1		
				Allow B1 for $0.6\,g - 0.2\,g = (0.6 + 0.2)a$ as an alternative for either of the above A marks
	Acceleration is 5 ms^{-2}	B1		
	Tension is 3 N	A1	5	
(ii)	$[0.9 = \frac{1}{2}\,5t^2]$	M1		For using $s = ut + \frac{1}{2}at^2$
	Time taken is 0.6 s	A1ft	2	ft $\sqrt{1.8/a}$

Question 2

(a) Car + trailer:	$2100a = 2380 - 280 - 630$	M1 A1
	$= 1470 \Rightarrow a = \underline{0.7\text{ m s}^{-2}}$	A1 (3)
(b) e.g. trailer:	$700 \times 0.7 = T - 280$	M1 A1√
	$\Rightarrow T = \underline{770\text{ N}}$	A1 (3)
(c) Car:	$1400a' = 2380 - 630$	M1 A1
	$\Rightarrow a' = 1.25\text{ m s}^{-2}$	A1
	distance $= 12 \times 4 + \frac{1}{2} \times 1.25 \times 4^2$	M1 A1√
	$= \underline{58\text{ m}}$	A1 (6)
(d) Same acceleration for car and trailer		B1 (1)

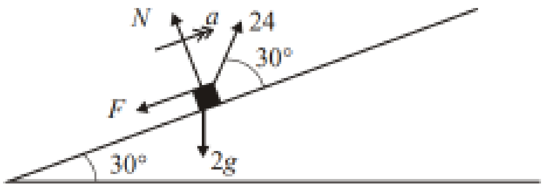
Forces - Slopes

Question 1

 (a) R (perp to plane): $R = 2g\cos 20$	M1 A1
$\approx$ <u>18.4 or 18 N</u>	A1 (3)
R (// to plane): $18 - 2g\sin 20 - F = 2a$	M1 A1
$F = 0.6R$ used	B1 ↓
Sub and solve: $a =$ <u>0.123 or 0.12 m s^{-2}</u>	M1 A1 (5)

Question 2

(a)



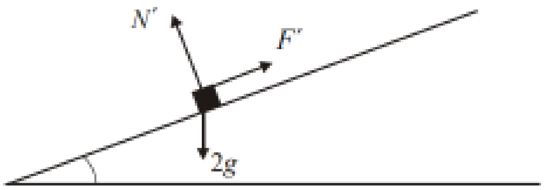
R(↖) $N + 24\cos 60° = 2g\cos 30°$ M1 A1 A1

$\Rightarrow N = 16.97 - 12 = 4.97$ N M1 A1

$\Rightarrow F = 0.4 \,.\, 4.97 = 1.99$ N M1 A1

R(↗) $2a = 24\cos 30° - 2g\cos 60° - 1.99$ A1 8

(b) $\Rightarrow a \approx 4.5$ m s^{-2}



R(↖) $N' = 2g\cos 30° = 16.97$ M1 A1

$\Rightarrow F'_{max} = 0.4 \,.\, 16.97 = 6.79$ N

Component of weight down plane = $2g\sin 30° = 9.8$ N M1

(c) $9.8 > F'_{max} \Rightarrow$ net force down plane $\Rightarrow$ parcel moves A1 4

$2f = 9.8 - 6.79, \Rightarrow f \approx 1.5$ m s^{-2} M1 A1, A1

[12]

Acceptable for find the acceleration in (b) to show this, then just give the value of a again in (c) with no further workings.