Y11 to Y12 Mathematics Summer Independent Learning

June to August 2024

Important notes:

1. **This is your Maths SIL for both Maths and Further Maths (Please do not do that regular Maths SIL)**
2. **This means there is (roughly) twice as much as your other two subjects, as it's two sets of lessons**

Please read the following instructions very carefully and ensure you label and collate all your work ready for checking in September.

For your first Maths lesson please bring

- A large A4 folder with five subject dividers.
- These instructions with the tables filled in (print out/copy the tables onto A4 paper).
- The two practice initial tests (Task 2), fully marked and reviewed.
- A list of questions you need to ask prior to doing your initial test.

Task 1: Preparation Work

1. Complete questions for each topic.
2. Mark and correct work.
3. Where required watch videos to support your understanding.

   Videos are listed after the intro to this task, and also within each topic

4. Do improvement work as necessary.
5. Repeat for each topic.
6. Track by filling in the following table.

Topic	Video(s) *(Tick)*	Worksheet *(Tick)*	Details of Improvement Work Completed
B1 Indices			
B2 Surds			
B3 Quadratics			
B4 Simultaneous Equations			
B5 Inequalities			
Re-arranging equations			
E1 Triangle Geometry			

Task 2

1. Do Practice Initial Test 1 under exam conditions.
2. Mark and correct your test and identify any improvement work necessary.
3. Fill in the review sheet below.

Topic	**Issues / areas for improvement (if relevant)**
B1 Indices	
B2 Surds	
B3 Quadratics	
B4 Simultaneous Equations	
B5 Inequalities	
Re-arranging equations	
E1 Triangle Geometry	

4. Do Practice Initial Test 2 under exam conditions.
5. Mark and correct your test and identify any improvement work necessary.
6. Fill in the review sheet below.
7. Make a list of questions you need to ask prior to doing your initial test for real!

Topic	**Issues / areas for improvement (if relevant)**
B1 Indices	
B2 Surds	
B3 Quadratics	
B4 Simultaneous Equations	
B5 Inequalities	
Re-arranging equations	
E1 Triangle Geometry	

Video hyperlinks

B1 Indices

https://youtu.be/1lThXgU08S0

https://youtu.be/v5bn4HZrmQs

https://youtu.be/W0h4rHj88ys

B2 Surds

https://youtu.be/jHelde32YtI

B3 Quadratics

https://youtu.be/Pziws8ojnlk

https://youtu.be/sn_joGVj15w

https://youtu.be/kk7p6hjn7hQ

https://youtu.be/toIqbX_NXHo

B4 Simultaneous Equations

https://youtu.be/4SRtwS5unwE

B5 Inequalities

https://youtu.be/wDut-In_7Wg

E1 Triangle Geometry

https://youtu.be/uVl6TAb0vBg

TASK 1

Indices and Surds

Topic: B1 Indices **Basic Skills videos:**
https://youtu.be/1lThXgU08S0
https://youtu.be/v5bn4HZrmQs
https://youtu.be/W0h4rHj88ys

Topic: B2 Surds Basic Skills
https://youtu.be/jHelde32YtI

Indices

Question 1

Express in the form x^k

a $\sqrt{x}$ **b** $\frac{1}{\sqrt[3]{x}}$ **c** $x^2 \times \sqrt{x}$ **d** $\frac{\sqrt[4]{x}}{x}$

e $\sqrt{x^3}$ **f** $\sqrt{x} \times \sqrt[3]{x}$ **g** $(\sqrt{x})^5$ **h** $\sqrt[3]{x^2} \times (\sqrt{x})^3$

i $p^{\frac{1}{4}} \div p^{-\frac{1}{5}}$ **j** $(3x^{\frac{2}{5}})^2$ **k** $y \times y^{\frac{5}{6}} \times y^{-\frac{3}{2}}$ **l** $4t^{\frac{3}{2}} \div 12t^{\frac{1}{2}}$

m $\frac{b^2 \times b^{\frac{1}{4}}}{b^{\frac{1}{2}}}$ **n** $\frac{y^{\frac{1}{2}} \times y^{\frac{1}{3}}}{y}$ **o** $\frac{4x^{\frac{2}{3}} \times 3x^{-\frac{1}{6}}}{6x^{\frac{3}{4}}}$ **p** $\frac{2a \times a^{\frac{3}{4}}}{8a^{-\frac{1}{2}}}$

Question 2

Express each of the following in the form 3^y, where y is a function of x.

a 9^x **b** 81^{x+1} **c** $27^{\frac{x}{4}}$ **d** $(\frac{1}{3})^x$ **e** 9^{2x-1} **f** $(\frac{1}{27})^{x+2}$

Question 3

Simplify in to one or more terms of ax^n (Where a and n are constants to be found, and not all questions use x)

a $\frac{x^3 + 2x}{x}$ **b** $\frac{4t^5 - 6t^3}{2t^2}$ **c** $\frac{x^{\frac{3}{2}} - 3x}{x^{\frac{1}{2}}}$ **d** $\frac{y^2(y^3 - 6)}{3y}$

e $\frac{p + p^{\frac{3}{2}}}{p^{\frac{3}{4}}}$ **f** $\frac{8w - 2w^{\frac{1}{2}}}{4w^{-\frac{1}{2}}}$ **g** $\frac{x + 1}{x^{\frac{1}{2}} + x^{-\frac{1}{2}}}$ **h** $\frac{2t^3 - 4t}{t^{\frac{3}{2}} - 2t^{-\frac{1}{2}}}$

Exam style question

Solve the equation

$$25^x = 5^{4x+1}.$$

Surds

Question 1

Simplify

a $\sqrt{18} + \sqrt{50}$ **b** $\sqrt{48} - \sqrt{27}$ **c** $2\sqrt{8} + \sqrt{72}$

Question 2

Express in the form $a + b\sqrt{3}$

a $\sqrt{3}(2 + \sqrt{3})$ **b** $4 - \sqrt{3} - 2(1 - \sqrt{3})$ **c** $(1 + \sqrt{3})(2 + \sqrt{3})$

Question 3

Express each of the following as simply as possible with a rational denominator.

a $\dfrac{1}{\sqrt{5}}$ **b** $\dfrac{2}{\sqrt{3}}$ **c** $\dfrac{1}{\sqrt{8}}$ **d** $\dfrac{14}{\sqrt{7}}$

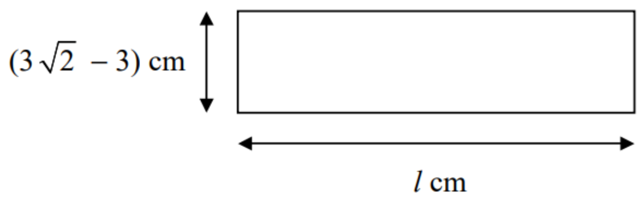
Question 4

Express each of the following as simply as possible with a rational denominator.

a $\dfrac{1}{\sqrt{2}+1}$ **b** $\dfrac{4}{\sqrt{3}-1}$ **c** $\dfrac{1}{\sqrt{6}-2}$ **d** $\dfrac{3}{2+\sqrt{3}}$

Exam style questions

(a)



The diagram shows a rectangle measuring $(3\sqrt{2} - 3)$ cm by l cm.
Given that the area of the rectangle is 6 cm^2, find the exact value of l in its simplest form.

(b)

Given that n is a positive integer, express

$$\frac{7}{3 + 5\sqrt{n}} - \frac{7}{5\sqrt{n} - 3}$$

as a single fraction not involving surds.

(c)

Solve the equation

$$3x = \sqrt{5}\,(x + 2),$$

giving your answer in the form $a + b\sqrt{5}$, where a and b are rational.

Indices answers

Question 1

a $= x^{\frac{1}{2}}$

b $= x^{-\frac{1}{3}}$

c $= x^2 \times x^{\frac{1}{2}} = x^{\frac{5}{2}}$

d $= \dfrac{x^{\frac{1}{4}}}{x} = x^{-\frac{3}{4}}$

e $= (x^3)^{\frac{1}{2}} = x^{\frac{3}{2}}$

f $= x^{\frac{1}{2}} \times x^{\frac{1}{3}} = x^{\frac{5}{6}}$

g $= (x^{\frac{1}{2}})^5 = x^{\frac{5}{2}}$

h $= x^{\frac{2}{3}} \times x^{\frac{3}{2}} = x^{\frac{13}{6}}$

i $= p^{\frac{1}{4} - (-\frac{1}{5})} = p^{\frac{9}{20}}$

j $= 9x^{\frac{4}{5}}$

k $= y^{1 + \frac{5}{6} - \frac{3}{2}} = y^{\frac{1}{3}}$

l $= \frac{1}{3}t$

m $= b^{2 + \frac{1}{4} - \frac{1}{2}} = b^{\frac{7}{4}}$

n $= y^{\frac{1}{2} + \frac{1}{3} - 1} = y^{-\frac{1}{6}}$

o $= 2x^{\frac{2}{3} + (-\frac{1}{6}) - \frac{3}{4}} = 2x^{-\frac{1}{4}}$

p $= \frac{1}{4}a^{1 + \frac{3}{4} - (-\frac{1}{2})} = \frac{1}{4}a^{\frac{9}{4}}$

Question 2

a $= (3^2)^x = 3^{2x}$

b $= (3^4)^{x+1} = 3^{4x+4}$

c $= (3^3)^{\frac{x}{4}} = 3^{\frac{3}{4}x}$

d $= (3^{-1})^x = 3^{-x}$

e $= (3^2)^{2x-1} = 3^{4x-2}$

f $= (3^{-3})^{x+2} = 3^{-3x-6}$

Question 3

a $= x^2 + 2$

b $= 2t^3 - 3t$

c $= x - 3x^{\frac{1}{2}}$

d $= \dfrac{y^5 - 6y^2}{3y}$

$= \frac{1}{3}y^4 - 2y$

e $= p^{\frac{1}{4}} + p^{\frac{3}{4}}$

f $= 2w^{\frac{3}{2}} - \frac{1}{2}w$

g $= \dfrac{x^{\frac{1}{2}}(x+1)}{x+1}$

$= x^{\frac{1}{2}}$

h $= \dfrac{t^{\frac{1}{2}} \times 2t(t^2 - 2)}{t^2 - 2}$

$= 2t^{\frac{3}{2}}$

Exam style question

$$25^x = (5^2)^x = 5^{4x+1}$$

$$5^{2x} = 5^{4x+1}$$

$$2x = 4x + 1$$

$$x = -\tfrac{1}{2}$$

Surds answers

Question 1

a $= 3\sqrt{2}+5\sqrt{2}=8\sqrt{2}$ **b** $= 4\sqrt{3}-3\sqrt{3}=\sqrt{3}$ **c** $= 4\sqrt{2}+6\sqrt{2}=10\sqrt{2}$

Question 2

a $= 3+2\sqrt{3}$

b $= 4-\sqrt{3}-2+2\sqrt{3}$
$= 2+\sqrt{3}$

c $= 2+\sqrt{3}+2\sqrt{3}+3$
$= 5+3\sqrt{3}$

Question 3

a $= \frac{1}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}} = \frac{1}{5}\sqrt{5}$ **b** $= \frac{2}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}} = \frac{2}{3}\sqrt{3}$ **c** $= \frac{1}{2\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}} = \frac{1}{4}\sqrt{2}$

d $= \frac{14}{\sqrt{7}}\times\frac{\sqrt{7}}{\sqrt{7}} = 2\sqrt{7}$

Question 4

$$\textbf{a} \quad = \frac{1}{\sqrt{2}+1}\times\frac{\sqrt{2}-1}{\sqrt{2}-1}=\frac{\sqrt{2}-1}{2-1} = \sqrt{2}-1$$

$$\textbf{b} \quad = \frac{4}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}=\frac{4(\sqrt{3}+1)}{3-1} = 2(\sqrt{3}+1)$$

$$\textbf{c} \quad = \frac{1}{\sqrt{6}-2}\times\frac{\sqrt{6}+2}{\sqrt{6}+2}=\frac{\sqrt{6}+2}{6-4} = \tfrac{1}{2}(\sqrt{6}+2) \text{ or } \tfrac{1}{2}\sqrt{6}+1$$

$$\textbf{d} \quad = \frac{3}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}=\frac{3(2-\sqrt{3})}{4-3} = 3(2-\sqrt{3})$$

Exam style questions

(a)

$$l = \frac{6}{3\sqrt{2}-3} = \frac{6}{3\sqrt{2}-3}\times\frac{3\sqrt{2}+3}{3\sqrt{2}+3} = \frac{6(3\sqrt{2}+3)}{18-9}$$

$$l = \frac{18(\sqrt{2}+1)}{9} = 2\sqrt{2}+2$$

(b)

$$\frac{7}{3+5\sqrt{n}}-\frac{7}{5\sqrt{n}-3}$$

$$=\frac{7\left(5\sqrt{n}-3\right)}{\left(3+5\sqrt{n}\right)\left(5\sqrt{n}-3\right)}-\frac{7\left(3+5\sqrt{n}\right)}{\left(3+5\sqrt{n}\right)\left(5\sqrt{n}-3\right)}$$

$$=\frac{35\sqrt{n}-21-\left(21+35\sqrt{n}\right)}{\left(3+5\sqrt{n}\right)\left(5\sqrt{n}-3\right)}$$

$$=-\frac{42}{25n-9}$$

(c)

$$3x = \sqrt{5}\,x + 2\sqrt{5}$$

$$x(3-\sqrt{5}) = 2\sqrt{5}$$

$$x = \frac{2\sqrt{5}}{3-\sqrt{5}} = \frac{2\sqrt{5}}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}} = \frac{2\sqrt{5}\,(3+\sqrt{5})}{9-5}$$

$$x = \frac{6\sqrt{5}+10}{4} = \frac{5}{2} + \frac{3}{2}\sqrt{5}$$

Quadratics, simultaneous equations and inequalities

Topic: B3 Quadratics **Basic Skills**
https://youtu.be/Pziws8ojnlk
https://youtu.be/sn_joGVj15w
https://youtu.be/kk7p6hjn7hQ https://youtu.be/tolqbX_NXHo +

B4 Simultaneous Equations
https://youtu.be/4SRtwS5unwE

B5 Inequalities
https://youtu.be/wDut-In_7Wg

Question 1

Factorise

(a)	x^2-3x+2	(b)	x^2+5x+6	(c)	x^2-9
(d)	$x^2-10x+25$	(e)	$2x^2-3x+1$	(f)	$5x^2-17x+6$
(g)	$16-9x^2$	(h)	x^4+4x^2+3	(i)	x^5-4x^3+4x

Question 2

Hence, sketch (showing the coordinates of any points of intersections with coordinate axes):

(a)	$y=x^2-3x+2$	(b)	$y=x^2+5x+6$	(c)	$y=x^2-9$
(d)	$y=x^2-10x+25$	(e)	$y=2x^2-3x+1$	(f)	$y=5x^2-17x+6$

Question 3

Complete the square, leaving in the form: $(x+a)^2+b$ or $a(x+b)^2+c$, where appropriate

(a)	x^2-4x+3	(b)	$x^2+8x+30$	(c)	x^2-5x+4
(d)	x^2+3x+3	(e)	$4x^2+8x+3$	(f)	$8+2x-x^2$

Question 4

Hence, sketch (showing the coordinates of turning point, and y intercept):

(a)	$y = x^2 - 4x + 3$	(b)	$y = x^2 + 8x + 30$	(c)	$y = x^2 - 5x + 4$
(d)	$y = x^2 + 3x + 3$	(e)	$y = 4x^2 + 8x + 3$	(f)	$y = 8 + 2x - x^2$

Exam style questions

(i)

a Express $x^2 - 4\sqrt{2}\,x + 5$ in the form $a(x + b)^2 + c$.

b Write down an equation of the line of symmetry of the curve $y = x^2 + 4\sqrt{2}\,x + 5$.

(ii)

$$f(x) \equiv x^2 + 2kx - 3.$$

By completing the square, find the roots of the equation $f(x) = 0$ in terms of the constant k.

(iii)

By completing the square, show that the roots of the equation $ax^2 + bx + c = 0$ are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Question 5

Solve these pairs of simultaneous equations:

(a)	$y = 2x + 6$ $y = 3 - 4x$	(b)	$3x + 3y + 4 = 0$ $5x - 2y - 5 = 0$	€	$x^2 - y + 3 = 0$ $x - y + 5 = 0$
(d)	$2x^2 - y - 8x = 0$ $x + y + 3 = 0$	(e)	$x^2 - 4y - y^2 = 0$ $x - 2y = 0$	(f)	$xy = 6$ $x - y = 5$
(g)	$\frac{3}{x} - 2y + 4 = 0$ $4x + y - 7 = 0$	(h)	$y = 2^x$ $4^x + y = 72$	(i)	$3^{x-1} = 9^{2y}$ $8^{x-2} = 4^{1+y}$

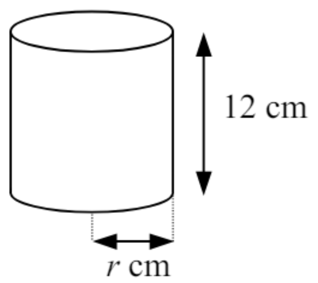
Question 6

Solve the following inequalities:

(a)	$12 - 3x < 10$	(b)	$2(3 + x) \geq 4(6 - x)$
€	$x^2 - 4x + 3 < 0$	(d)	$9x - 2x^2 \leq 10$

Exam style questions

(a)



A sealed metal container for food is a cylinder of height 12 cm and base radius r cm.

Given that the surface area of the container must be at most 128π cm^2,

a show that $r^2 + 12r - 64 \leq 0$.

b Hence find the maximum value of r.

(b)

The cost for framing a picture is

- 2 pence per cm^2 of glass.
- 5 pence per cm of wooden frame.

A rectangular picture is such so that its length is 4 cm greater than its width, $x\ cm$.

If a **maximum** of £10 is available for framing, determine the range of the possible values of x.

Quadratics, simultaneous equations and inequalities answers

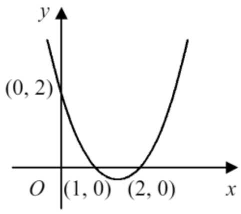
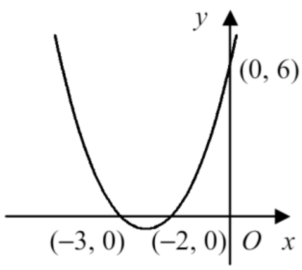
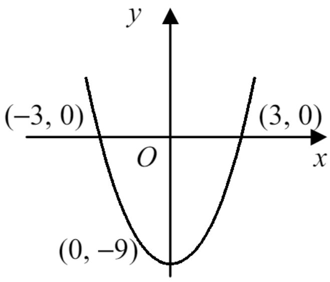
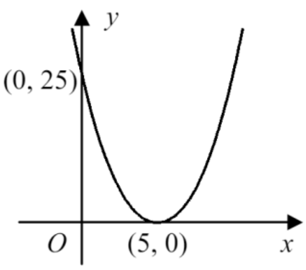
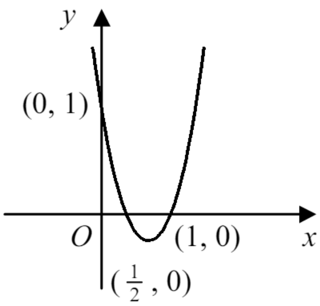
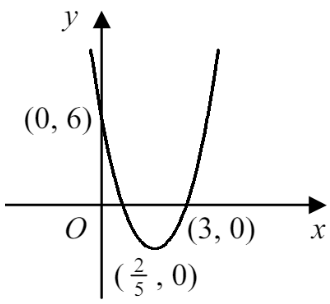
Question 1

Factorise

(a)	$(x-1)(x-2)$	(b)	$(x+3)(x+2)$	€	$(x+3)(x-3)$
(d)	$(x-5)^2$	€	$(2x-1)(x-1)$	(f)	$(5x-2)(x-3)$
(g)	$(4+3x)(4-3x)$	(h)	$(x^2+3)(x^2+1)$	(i)	$x(x^4-4x^2+4)$ $x(x^2-2)^2$

Question 2

Hence, sketch (showing the coordinates of any points of intersections with coordinate axes):

(a)	$x^2-3x+2=0$ $(x-1)(x-2)=0$ $x=1$ or 2 	(b)	$x^2+5x+6=0$ $(x+3)(x+2)=0$ $x=-3$ or -2 	€	$x^2-9=0$ $(x+3)(x-3)=0$ $x=-3$ or 3 
(d)	$x^2-10x+25=0$ $(x-5)^2=0$ $x=5$ 	€	$2x^2-3x+1=0$ $(2x-1)(x-1)=0$ $x=\frac{1}{2}$ or 1 	(f)	$5x^2-17x+6=0$ $(5x-2)(x-3)=0$ $x=\frac{2}{5}$ or 3 

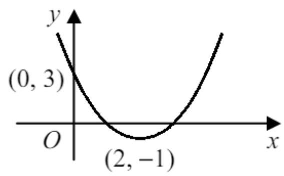
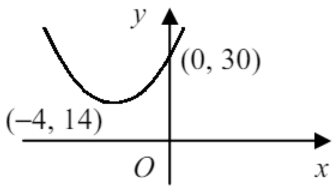
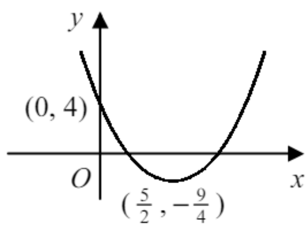
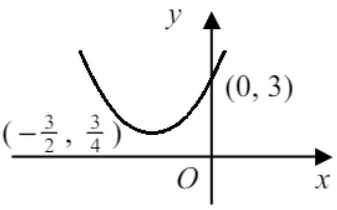
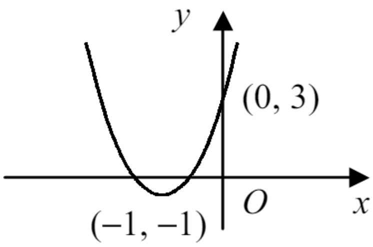
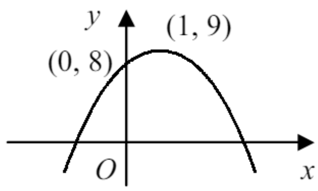
Question 3

Complete the square, leaving in the form: $(x+a)^2+b$ or $a(x+b)^2+c$, where appropriate

(a)	$y=(x-2)^2-4+3$ $y=(x-2)^2-1$	(b)	$y=(x+4)^2-16+30$ $y=(x+4)^2+14$	€	$y=(x-\frac{5}{2})^2-\frac{25}{4}+4$ $y=(x-\frac{5}{2})^2-\frac{9}{4}$
(d)	$y=(x+\frac{3}{2})^2-\frac{9}{4}+3$ $y=(x+\frac{3}{2})^2+\frac{3}{4}$	(e)	$y=4[x^2+2x]+3$ $y=4[(x+1)^2-1]+3$ $y=4(x+1)^2-1$	(f)	$y=-[x^2-2x]+8$ $y=-[(x-1)^2-1]+8$ $y=-(x-1)^2+9$

Question 4

Hence, sketch (showing the coordinates of turning point, and y intercept):

(a)	$y=(x-2)^2-4+3$ $y=(x-2)^2-1$ minimum $(2,-1)$ 	(b)	$y=(x+4)^2-16+30$ $y=(x+4)^2+14$ minimum $(-4, 14)$ 	€	$y=(x-\frac{5}{2})^2-\frac{25}{4}+4$ $y=(x-\frac{5}{2})^2-\frac{9}{4}$ minimum $(\frac{5}{2},-\frac{9}{4})$ 
(d)	$y=(x+\frac{3}{2})^2-\frac{9}{4}+3$ $y=(x+\frac{3}{2})^2+\frac{3}{4}$ minimum $(-\frac{3}{2},\frac{3}{4})$ 	€	$y=4[x^2+2x]+3$ $y=4[(x+1)^2-1]+3$ $y=4(x+1)^2-1$ minimum $(-1,-1)$ 	(f)	$y=-[x^2-2x]+8$ $y=-[(x-1)^2-1]+8$ $y=-(x-1)^2+9$ maximum $(1, 9)$ 

Exam style questions

(i)

a $= (x - 2\sqrt{2})^2 - 8 + 5$

$= (x - 2\sqrt{2})^2 - 3$

b $x = 2\sqrt{2}$

(ii)

$$x^2 + 2kx - 3 = 0$$
$$(x + k)^2 - k^2 - 3 = 0$$
$$(x + k)^2 = k^2 + 3$$
$$x + k = \pm\sqrt{k^2 + 3}$$
$$x = -k \pm \sqrt{k^2 + 3}$$

(iii)

$$ax^2 + bx + c = 0$$
$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$
$$(x + \frac{b}{2a})^2 - \frac{b^2}{4a^2} + \frac{c}{a} = 0$$
$$(x + \frac{b}{2a})^2 = \frac{b^2}{4a^2} - \frac{c}{a} = \frac{b^2 - 4ac}{4a^2}$$
$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$
$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Question 5

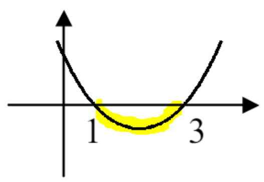
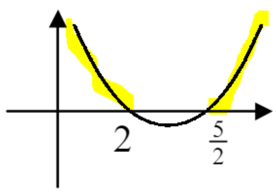
Solve these pairs of simultaneous equations:

(a)	$2x + 6 = 3 - 4x$ $x = -\frac{1}{2}$ $\therefore\ x = -\frac{1}{2}, y = 5$	(b)	$6x + 6y + 8 = 0$ $15x - 6y - 15 = 0$ adding $21x - 7 = 0$ $x = \frac{1}{3}$ $\therefore\ x = \frac{1}{3}, y = -\frac{5}{3}$	(c)	$x + 2 = x^2 - 4$ $x^2 - x - 6 = 0$ $(x + 2)(x - 3) = 0$ $x = -2$ or 3 $\therefore\ (-2, 0)$ and $(3, 5)$

(d)	**Susbtitution is also fine** adding $2x^2 - 7x + 3 = 0$ $(2x - 1)(x - 3) = 0$ $x = \frac{1}{2}$ or 3 $\therefore \quad x = \frac{1}{2}, y = -\frac{7}{2}$ or $\quad x = 3, y = -6$	(e)	$x = 2y$ sub. $(2y)^2 - 4y - y^2 = 0$ $3y^2 - 4y = 0$ $y(3y - 4) = 0$ $y = 0$ or $\frac{4}{3}$ $\therefore \quad x = 0, y = 0$ or $\quad x = \frac{8}{3}, y = \frac{4}{3}$	(f)	$y = x - 5$ sub. $x(x - 5) = 6$ $x^2 - 5x - 6 = 0$ $(x + 1)(x - 6) = 0$ $x = -1$ or 6 $\therefore \quad x = -1, y = -6$ or $\quad x = 6, y = 1$
(g)	$y = 7 - 4x$ sub. $\frac{3}{x} - 2(7 - 4x) + 4 = 0$ $3 - 2x(7 - 4x) + 4x = 0$ $8x^2 - 10x + 3 = 0$ $(4x - 3)(2x - 1) = 0$ $x = \frac{1}{2}$ or $\frac{3}{4}$ $\therefore \quad x = \frac{1}{2}, y = 5$ or $\quad x = \frac{3}{4}, y = 4$	(h)	$4^x + 2^x = 72$ $(2)^{2x} + 2^x - 72 = 0$ $(2^x + 9)(2^x - 8) = 0$ $2^x \neq -9, 2^x = 8$ $x = 3$ $y = 8$	(i)	$3^{x-1} = (3^2)^{2y} \quad \therefore\ x - 1 = 4y$ $(2^3)^{x-2} = (2^2)^{1+y} \quad \therefore\ 3x - 6 = 2 + 2y$ $6x - 16 = 4y$ $\Rightarrow \quad 6x - 16 = x - 1$ $x = 3$ $\therefore \quad x = 3, y = \frac{1}{2}$

Question 6

Solve the following inequalities:

(a)	$2 < 3x$ $x > \frac{2}{3}$	(b)	$6 + 2x \geq 24 - 4x$ $6x \geq 18$ $x \geq 3$
(c)	$(x - 1)(x - 3) < 0$  $\therefore\ 1 < x < 3$	(d)	$2x^2 - 9x + 10 \geq 0$ $(2x - 5)(x - 2) \geq 0$  $\therefore\ x \leq 2$ or $x \geq \frac{5}{2}$

Exam style question

(i)

a $\text{S.A} = 2\pi r^2 + 2\pi rh = 2\pi r^2 + 24\pi r$

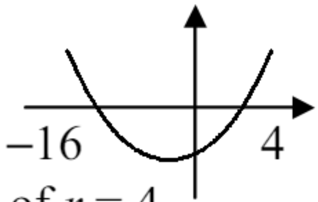
$\text{S.A} \le 128\pi \quad \therefore \quad 2\pi r^2 + 24\pi r \le 128\pi$

$$r^2 + 12r \le 64$$

$$r^2 + 12r - 64 \le 0$$

b $(r + 16)(r - 4) \le 0$

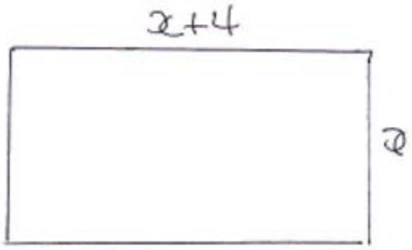
$-16 \le r \le 4$



$\therefore$ maximum value of $r = 4$

We will look at finding maximum values for these kinds of shapes more formally in A level Maths

(ii)



- $P = 2x + 2(x+4)$
  $P = (4x+8)$ cm
- $A = x(x+4)$
  $A = (x^2+4x)$ cm^2

COST (IN PENCE) $= 5(4x+8) + 2(x^2+4x)$

$= 20x + 40 + 2x^2 + 8x$

$= 2x^2 + 28x + 40$

THUS $2x^2 + 28x + 40 < 1000$ £10

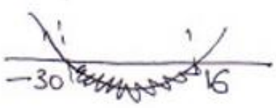
$\Rightarrow 2x^2 + 28x - 960 < 0$

$\Rightarrow x^2 + 14x - 480 < 0$

$\Rightarrow (x-16)(x+30) < 0$

6×80
12×40
24×20
16×30

C.V $= \begin{cases} 16 \\ -30 \end{cases} \Rightarrow$



$\Rightarrow \therefore 0 < x < 16$

$-30 < x < 16$

Re-arranging (Equations and formulae)

Question 1

Make a the subject $x(a - e) = d$

Question 2

Make x the subject $m(y - x) = t$

Question 3

Make x the subject of $x + a = \frac{x+b}{c}$

Question 4

Make y the subject of $y(\sqrt{3} + \sqrt{2}) = x$

and write it in the form $y = x(\sqrt{a} + \sqrt{b})$

Question 5

Make v the subject of

$$C = \frac{v^2 - ta}{x}$$

Question 6

Rearrange to make x the subject of

$$\frac{2}{x} + 5 = 6y$$

Question 7

Make y the subject of

$$\sqrt{\frac{m(y+a)}{y}} = g$$

Question 8

A cylinder has a radius of 3cm and height, h. The total surface area is $30x\ cm^2$.

Find an expression for the surface area and write h in terms of x and π.

Prove it questions

Using your rearranging skills can you prove each of the following

If $\boldsymbol{a = \frac{b}{b+c}}$

Show that $\boldsymbol{\frac{a}{1-a} = \frac{b}{c}}$

$\frac{n(n-1)}{2} + \frac{n(n+1)}{2}$ is a square number

$$\frac{2x+3}{4} - \frac{3x-2}{3} + \frac{1}{6} = \frac{19-6x}{12}$$

Re-arranging (Equations and formulae)

Question 1

$$xa - xe = d$$
$$xa = d + xe$$
$$a = \frac{d + xe}{x}$$

or

$$a - e = \frac{d}{x}$$
$$a = \frac{d}{x} + e$$

Can you see that these are equivalent?

Question 2

$$my - mx = t$$
$$my = t + mx$$
$$mx = my - t$$
$$x = \frac{my - t}{m}$$

Question 3

$$c(x + a) = x + b$$
$$cx + ca - x = b$$
$$cx - x = b - ca$$
$$x(c - 1) = b - ca$$
$$x = \frac{b - ca}{c - 1}$$

Question 4

$$y = \frac{x}{\sqrt{3}+\sqrt{2}}$$

$$y = \frac{x}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

$$y = \frac{x\sqrt{3} - x\sqrt{2}}{3-2}$$

$$y = x(\sqrt{3}-\sqrt{2})$$

Question 5

$$v^2 - ta = Cx$$

$$v^2 = Cx + ta$$

$$v = \pm\sqrt{Cx + ta}$$

Question 6

$$\frac{2}{x} = 6y - 5$$

$$x(6y-5) = 2$$

$$x = \frac{2}{6y-5}$$

Question 7

$$g^2 = \frac{my + ma}{y}$$
$$g^2y = my + ma$$
$$g^2y - my = ma$$
$$y(g^2 - m) = ma$$
$$y = \frac{ma}{g^2 - m}$$

Question 8

Surface area of cylinder = $2\pi r^2 + 2\pi rh$

$$30x = (2\pi \times 3^2) + (2 \times 3 \times \pi \times h)$$
$$30x = 18\pi + 6\pi h$$
$$6\pi h = 30x - 18\pi$$
$$h = \frac{30x - 18\pi}{6\pi}$$
$$h = \frac{5x - 3\pi}{\pi}$$

Prove it solutions

If $a = \frac{b}{b+c}$

Show that $\frac{a}{a-1} = \frac{b}{c}$

$\frac{a}{1} = \frac{b}{b+c}$	Make a into a fraction
$a(b+c) = b$	Using what we know about the product of the diagonals of equivalent fractions
$ab + ac = b$	Expand brackets
$ac = b - ab$	Make ac the subject
$ac = b(1-a)$	Factorise the right hand side
$b = \frac{ac}{1-a}$	Make b the subject
$\frac{b}{c} = \frac{a}{1-a}$	Divide both sides by c, expression as required

$$\boxed{\frac{n(n-1)}{2} + \frac{n(n+1)}{2} \quad \text{is a square number}}$$

$$\frac{n^2-n}{2} + \frac{n^2+n}{2}$$

Expand brackets

$$\frac{n^2-n+n^2+n}{2}$$

Write as one fraction

$$\frac{2n^2}{2}$$

Simplify numerator

$$n^2$$

Cancel out factor of 2 so left with n^2 which is a square number as required

$$\boxed{\frac{2x+3}{4} - \frac{3x-2}{3} + \frac{1}{6} = \frac{19-6x}{12}}$$

$$\frac{2x+3}{4} - \frac{3x-2}{3} + \frac{1}{6}$$

Concentrate on Left hand side

X 3 X 4 X 2

$$\frac{3(2x+3)}{12} - \frac{4(3x-2)}{12} + \frac{2}{12}$$

Make a common denominator

$$\frac{6x+9}{12} - \frac{12x-8}{12} + \frac{2}{12}$$

Expand brackets

$$\frac{6x+9-(12x-8)+2}{12}$$

Collect terms over single denominator

$$\frac{6x+9-12x+8+2}{12}$$

Simplify

$$\frac{19-6x}{12}$$

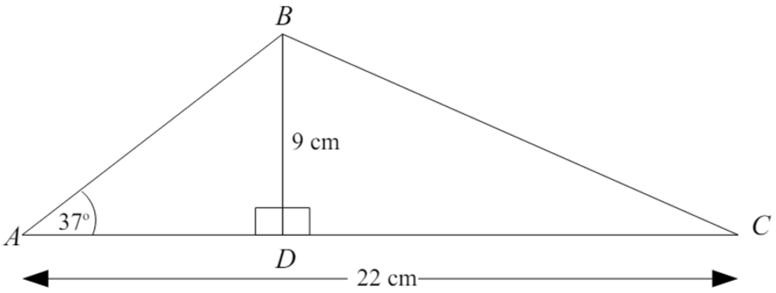
Left hand side is = to right hand side as required

Trigonometry

E1 Triangle Geometry

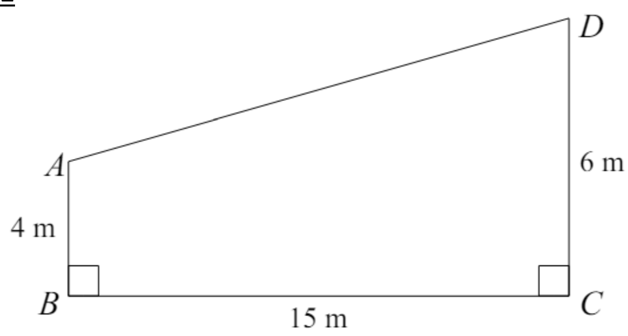
https://youtu.be/uVl6TAb0vBg

Question 1



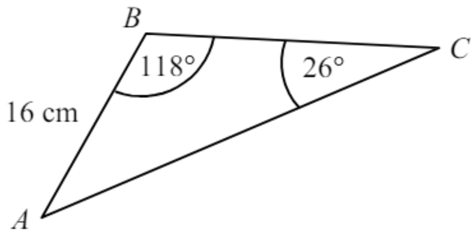
Work out the size of angle BCD.
Give your answer to 1 decimal place.

Question 2



Work out the size of angle BAD.
Give your answer to 1 decimal place.

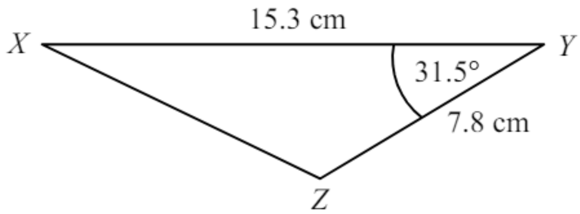
Question 3



The diagram shows triangle ABC in which $AB = 16$ cm, $\angle ABC = 118°$ and $\angle ACB = 26°$.

Find the length AC to 3 significant figures.

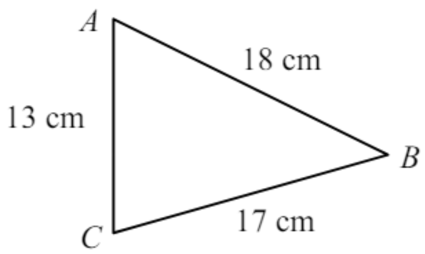
Question 4



The diagram shows triangle XYZ in which $XY = 15.3$ cm, $YZ = 7.8$ cm and $\angle XYZ = 31.5°$.

Find the length of XZ.

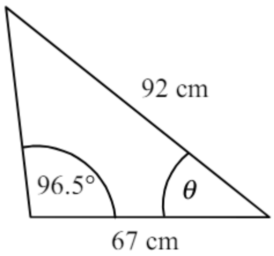
Question 5



The diagram shows triangle ABC in which $AB = 18$ cm, $AC = 13$ cm and $BC = 17$ cm.

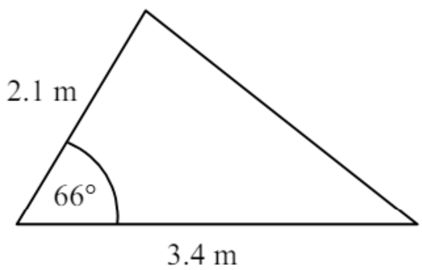
Find the size of the angle ACB

Question 6



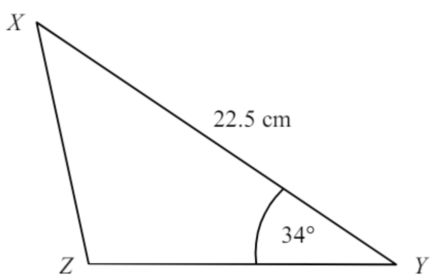
Find the angle θ

Question 7



Find the area of the triangle

Question 8



The diagram shows triangle XYZ in which $XY = 22.5$ cm and $\angle XYZ = 34°$.

Find the length of XZ

Supporting guidance – if needed

You can of course get one solution to an equation such as $\sin x = -0.5$ from your calculator. But what about others?

Example 1 Solve the equation $\sin x° = -0.5$ for $0 \leq x < 360$.

Solution The calculator gives $\sin^{-1}(0.5) = -30$.

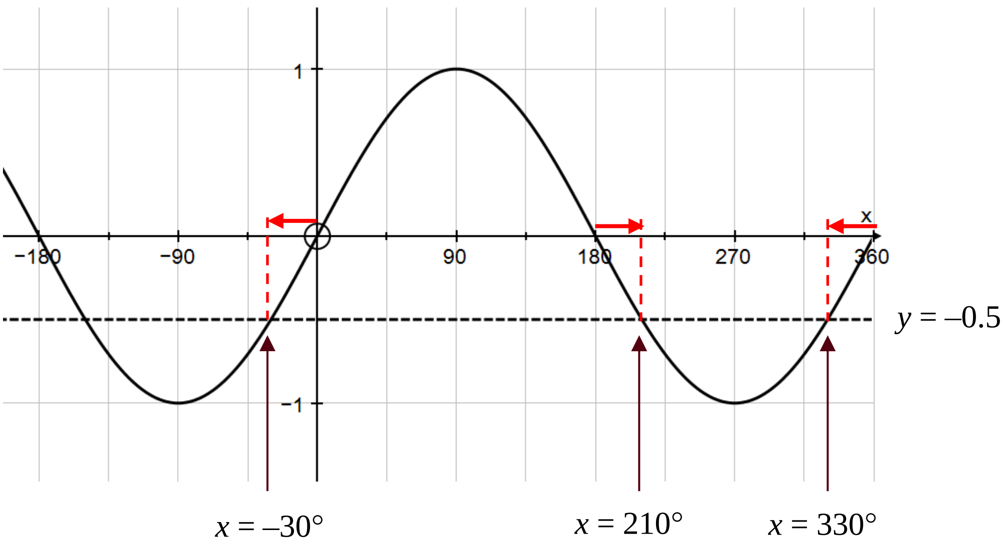
This is usually called the *principal value* of the function $\sin^{-1}$.

To get a second solution you can either use a graph or a standard rule.

Method 1: Use the graph of $y = \sin x$

By drawing the line $y = -0.5$ on the same set of axes as the graph of the sine curve, points of intersection can be identified in the range

$0 \leq x < 360$.



(The red arrows each indicate 30° to one side or the other.)

Hence the required solutions are 210° or 330°.

Method 2: Use an algebraic rule.

To find the second solution you use $\sin (180 - x)° = \sin x°$

$\tan (180 + x)° = \tan x°$

$\cos (360 - x)° = \cos x°$.

Any further solutions are obtained by adding or subtracting 360 from the principal value or the second solution.

In this example the principal solution is $-30°$.

Therefore, as this equation involves sine, the second solution is:

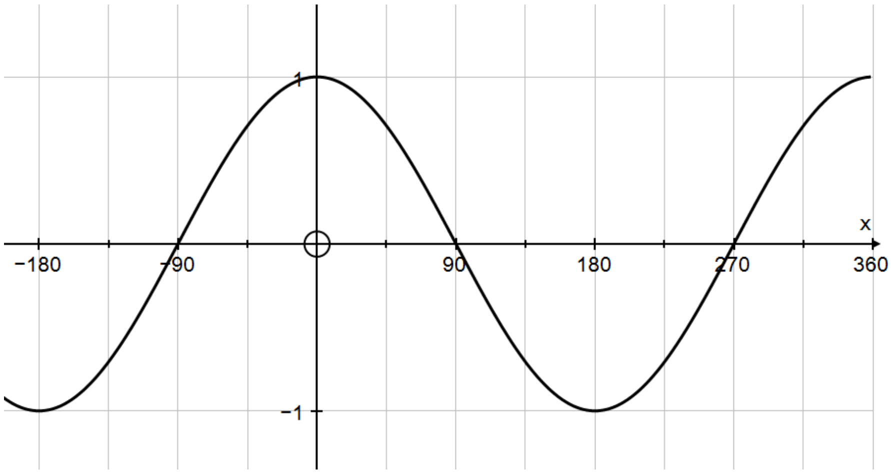
$$180 - (-30)° = 210°$$

$-30°$ is not in the required range, so add 360 to get:

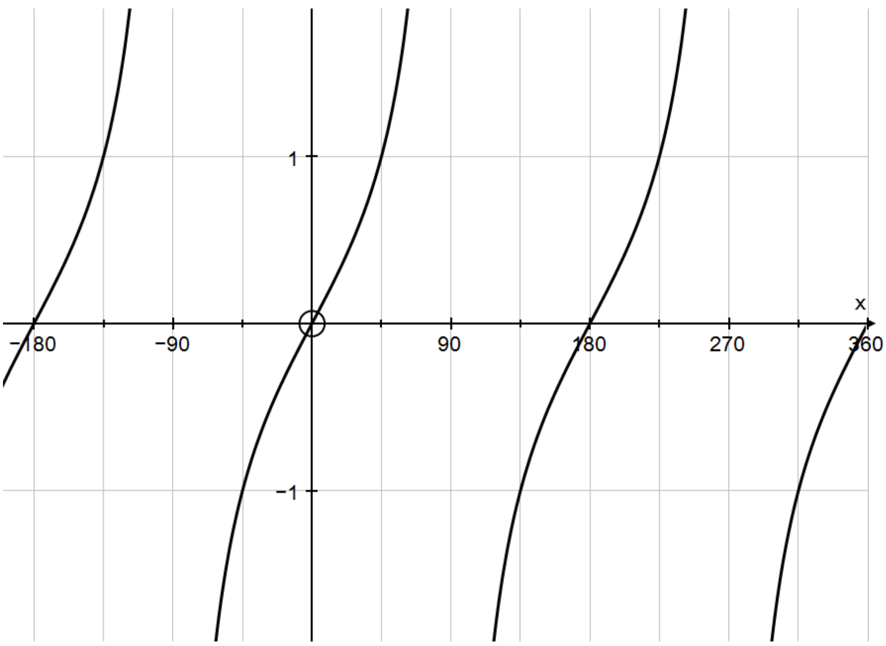
$$360 + (-30) = 330°.$$

Hence the required solutions are $210°$ or $330°$.

You should decide which method you prefer. The corresponding graphs for $\cos x$ and $\tan x$ are shown below.



$y = \cos x$



$y = \tan x$

To solve equations of the form $y = \sin(kx)$, you will expect to get $2k$ solutions in any interval of $360°$. You can think of compressing the graphs, or of using a wider initial range.

Example 2 Solve the equation $\sin 3x° = 0.5$ for $0 \le x < 360$.

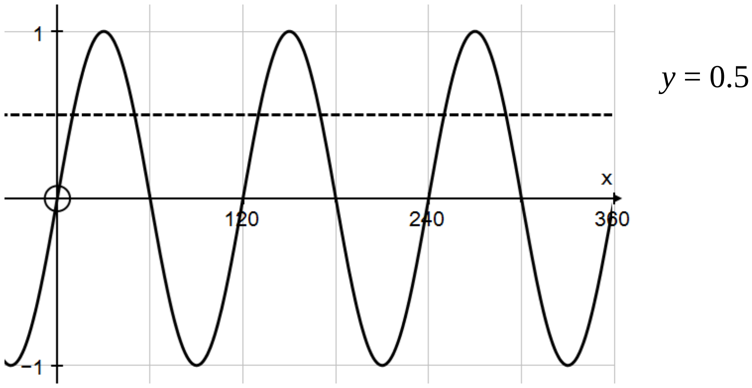
Solution ***Method 1:*** Use the graph.

The graph of $y = \sin 3x°$ is the same as the graph of $y = \sin x°$ but compressed by a factor of 3 (the *period* is $120°$).

The calculator gives $\sin^{-1}(0.5) = 30$, so the principal solution is given by

$$3x = 30 \Rightarrow x = 10.$$

The vertical lines on the graph below are at multiples of $60°$. So you can see from the graph that the other solutions are $50°$, $130°$, $170°$, $250°$ and $290°$.



$y = \sin 3x$

Method 2: The principal value of $3x$ is $\sin^{-1}(0.5) = 30°$.

Therefore $3x = 30$ or $180 - 30 = 150$,

or $360 + 30$ or $360 + 150$

or $720 + 30$ or $720 + 150$

$\Rightarrow 3x = 30, 150, 390, 510, 750, 870$

$\Rightarrow x = 10, 50, 130, 170, 250, 290.$

Notice that with Method 2 you have to look at values of $3x$ in the range 0 to 1080 ($= 3 \times 360$), which is somewhat non-intuitive.

Question 1

Solve the following equations for $0 \le x < 360$. Give your answers to the nearest $0.1°$.

(a) $\sin x° = 0.9$ (b) $\cos x° = 0.6$ (c) $\tan x° = 2$

(d) $\sin x° = -0.4$ (e) $\cos x° = -0.5$ (f) $\tan x° = -3$

Question 2

Solve the following equations for $-180 \le x < 180$. Give your answers to the nearest $0.1°$.

(a) $\sin x° = 0.9$ (b) $\cos x° = 0.6$ (c) $\tan x° = 2$

(d) $\sin x° = -0.4$ (e) $\cos x° = -0.5$ (f) $\tan x° = -3$

Question 3

Solve the following equations for $0 \le x < 360$. Give your answers to the nearest $0.1°$.

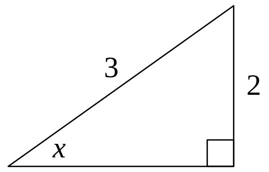
(a) $\sin 2x° = 0.829$ (b) $\cos 3x° = 0.454$ (c) $\tan 4x = 2.05$

(d) $\sin \frac{1}{2}x° = 0.8$ (e) $\cos \frac{1}{2}x° = 0.3$ (f) $\tan \frac{1}{3}x° = 0.7$

E3 Exact Trigonometric values

Supporting guidance – if needed

Suppose that you are told that $\sin x°$ is exactly $\frac{2}{3}$. Assuming that x is between $0°$ and $90°$, you can find the exact values of $\cos x°$ and $\tan x°$ by drawing a right-angled triangle in which the opposite side and the hypotenuse are 2 and 3 respectively:



Now Pythagoras's Theorem tells you that the third, adjacent, side is $\sqrt{3^2 - 2^2} = \sqrt{5}$.

Hence using SOH, CAH, TOAH, $\cos x°$ = $\dfrac{\sqrt{5}}{3}$ and $\tan x°$ = $\dfrac{2}{\sqrt{5}}$.

This is preferable to using a calculator as the calculator does not always give exact values for this type of calculation. (Calculators can *in general* not handle irrational numbers exactly, although many are programmed to do so in simple cases.)

Question 1

Do not use a calculator in this exercise.

In this question θ is in the range $0 \leq \theta < 90$.

(a) Given that $\sin\theta = \dfrac{12}{13}$, find the exact values of $\cos\theta$ and $\tan\theta$.

(b) Given that $\tan\theta = \dfrac{6}{7}$, find the exact values of $\sin\theta$ and $\cos\theta$.

(c) Given that $\cos\theta = \dfrac{5}{8}$, find the exact values of $\sin\theta$ and $\tan\theta$.

Trigonometry answers

Triangle geometry

<u>Question 1</u>

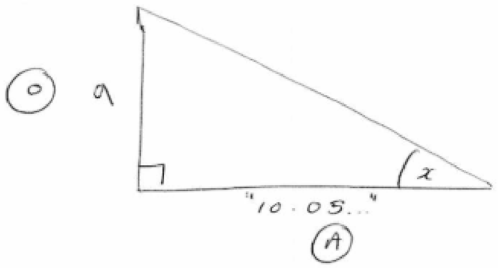
$$\tan(37) = \frac{9}{y}$$

$$y = \frac{9}{\tan(37)}$$

$$= 11.9434\ldots$$

$$CD = 22 - 11.9434$$

$$= 10.05659\ldots$$



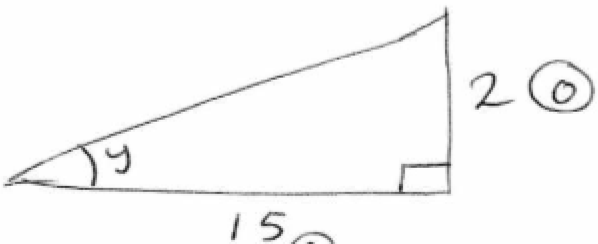
$$\tan x = \frac{9}{\text{"}10.05..\text{"}}$$

$$x = \tan^{-1}\left(\frac{9}{10.05\ldots}\right)$$

$$= 41.8 \quad 1dp$$

41.8 °

<u>Question 2</u>



$$\tan y = \frac{2}{15}$$

$$y = \tan^{-1}\left(\frac{2}{15}\right)$$

$$= \underline{\underline{7.6^\circ}} \quad (1dp)$$

$$BAD = 90 + 7.6$$

$$= \underline{\underline{97.6^\circ}}$$

Question 3

$$\frac{AC}{\sin 118} = \frac{16}{\sin 26}$$
$$AC = \frac{16 \times \sin 118}{\sin 26}$$
$$= 32.2 \text{ cm}$$

Question 4

$$XZ^2 = 7.8^2 + 15.3^2 \quad - (2 \times 7.8 \times 15.3 \times \cos 31.5°)$$

$$= 91.422$$
$$XZ = 9.56 \text{ cm (3sf)}$$

Question 5

$$18^2 = 13^2 + 17^2 - (2 \times 13 \times 17 \times \cos \angle ACB)$$
$$\cos \angle ACB = \frac{13^2 + 17^2 - 18^2}{2 \times 13 \times 17}$$
$$= 0.3032$$
$$\angle ACB = 72.4° \text{ (1dp)}$$

Question 6

$$\frac{\sin \alpha}{67} = \frac{\sin 96.5}{92}$$
$$\sin \alpha = \frac{67 \times \sin 96.5}{92}$$

$$\sin \alpha = 0.7236$$

$$\alpha = 46.351$$

$$\theta = 180 - 96.5 - \alpha$$

$$\theta = 37.1° \text{ (1dp)}$$

Question 7

area

$$= \tfrac{1}{2} \times 2.1 \times 3.4 \times \sin 66$$
$$= 3.26 \text{ m}^2 \text{ (3sf)}$$

Question 8

$$\tfrac{1}{2} \times 22.5 \times YZ \times \sin 34 = 100$$
$$YZ = \frac{200}{22.5 \times \sin 34}$$
$$= 15.896$$

$$XZ^2 = 22.5^2 + 15.896^2 - (2 \times 22.5 \times 15.896 \times \cos 34)$$
$$= 165.906$$
$$XZ = 12.9 \text{ cm (3sf)}$$

Trigonometric equations

Question 1

(a) 64.2, 115.8 (b) 53.1, 306.9 (c) 63.4, 243.4

(d) 203.6, 336.4 (e) 120, 240 (f) 108.4, 288.4

Question 2

(a) 64.2, 115.8 (b) 53.1, –53.1 (c) 63.4, –116.6

(d) –23.6, –156.4 (e) 120, –120 (f) –71.5, 108.4

Question 3

(a) 28, 62, 208, 242 (b) 21, 99, 141, 219, 261, 339

(c) 16, 61, 106, 151, 196, 241, 286, 331 (d) 106.2, 253.7

(e) 145.1 (f) 105

Exact Trigonometric values

Question 1

(a) $\frac{5}{13}, \frac{12}{5}$ (b) $\frac{6}{\sqrt{85}}, \frac{7}{\sqrt{85}}$ (c) $\frac{\sqrt{39}}{8}, \frac{\sqrt{39}}{5}$

TASK 2

Year 12 Initial Test for Mathematics

*Write out the solutions to each of the following questions. Show full working, **without** the use of a calculator.*

Practice 1

B1 Indices

1.	Evaluate $\left(\frac{8}{125}\right)^{-2/3}$	**2.**	Express in the form x^k $\frac{\sqrt{x} \times \sqrt[3]{x}}{x^2}$	**3.**	Solve $9^{x-2} = 27$	**4.**	Solve $16^x = 4^{1-x}$

B2 Surds

1.	Simplify $\sqrt{72}$	**2.**	Expand and simplify $(2\sqrt{7} - 5\sqrt{3})(3\sqrt{7} + 4\sqrt{3})$	**3.**	Rationalise the denominator $\frac{11}{2\sqrt{5}}$	**4.**	Rationalise the denominator $\frac{8 - 3\sqrt{5}}{2 + \sqrt{5}}$

B3 Quadratics

1. Solve the following quadratic equations by factorising and use your solutions to sketch the related quadratic graph, labelling all intersections with the coordinate axis.

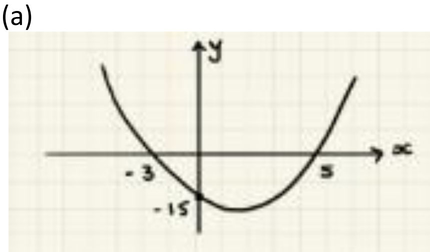
(a) (i) $x^2 + 3x - 28 = 0$	(b) (i) $x^2 - 6x + 9 = 0$	(c) (i) $2x^2 - 21x + 27 = 0$
(a) (ii) Sketch $y = x^2 + 3x - 28$	(b) (ii) Sketch $y = x^2 - 6x + 9$	(c) (ii) Sketch $y = 2x^2 - 21x + 27$

2. Solve the following quadratic equations by completing the square and use your solutions to sketch the related quadratic graph, labelling all intersections with the coordinate axis and turning point.

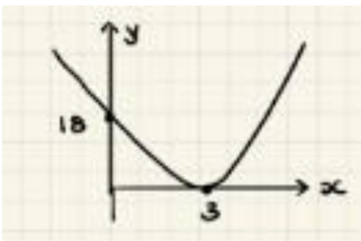
(a) (i) $x^2 + 4x - 7 = 0$	(b) (i) $11 + 8x - x^2 = 0$	(c) (i) $3x^2 - 12x + 2 = 0$
(ii) Write $y = x^2 + 4x - 7$ in the form $y = a(x+b)^2 + c$	(ii) Write $y = 11 + 8x - x^2$in the form $y = a(x+b)^2 + c$	(ii) Write $y = 3x^2 - 12x + 2$ in the form $y = a(x+b)^2 + c$
(iii) Sketch $y = x^2 + 4x - 7$	(iii) Sketch $y = 11 + 8x - x^2$	(iii) Sketch $y = 3x^2 - 12x + 2$

3. Evaluate the equation of the following quadratics, giving your answer in the form $y = ax^2 + bx + c$

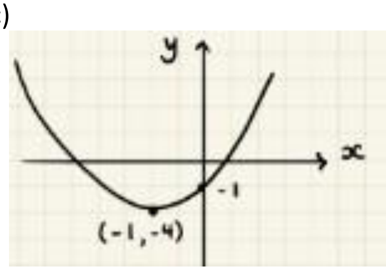
(a)



(b)



(c)



B4 Simultaneous Equations

1.	Solve $3x + 3y = -4$ $5x - 2y = 5$	**2.**	Solve $y = x - 6$ $\frac{1}{2}x - y = 4$	**3.**	Solve $3x^2 - x - y^2 = 0$ $x + y = 1$

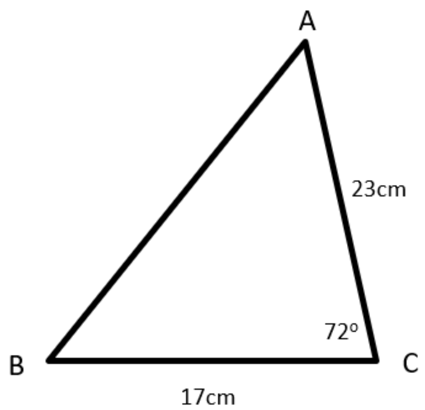
B5 Inequalities

Find the set of values for which…

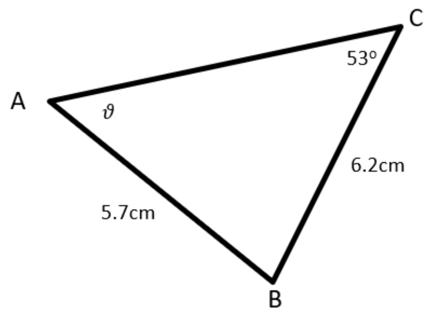
1.	$3(1 - 2t) \leq t - 4$	**2.**	$2x^2 - 9x + 4 \leq 0$	**3.**	$2y + 3 < 3y(y - 2)$

E1 Triangle Geometry (Calculator)

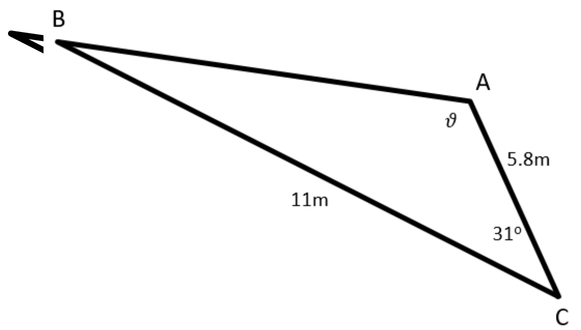
1. Calculate the length AB



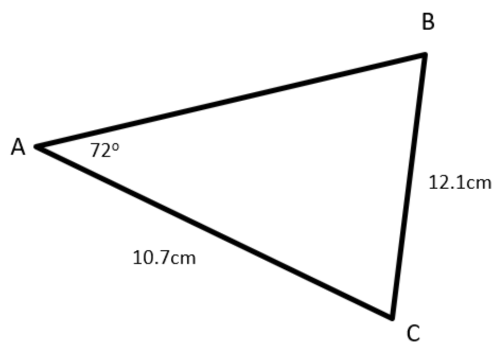
2. Calculate the angle ϑ



3. Calculate the length AB and the obtuse angle ϑ



4. Calculate the area of the triangle ABC



Re-arranging equations

1.	To find velocity, v, we use the formula $v^2 = u^2 + 2as$ Rearrange to find s	**2.**	Make x the subject of $4F = F + \dfrac{a}{y + x}$

E7 Trigonometric equations

Solve each equation for θ in the interval $0 \le \theta \le 360°$ giving your answers to 1 decimal place.			
1.	$\cos\theta = 0.4$	**2.**	$\sin 2x° = 0.5$

E3 Exact Trigonometric values

Find the exact values of $\cos x$ and $\tan x$ given that:

$\sin x = \dfrac{4}{5}$ and $0^\circ < x < 90^\circ$

Practice 1

B1 Indices

1. $\left(\frac{8}{125}\right)^{-2/3}$

$= \left(\frac{125}{8}\right)^{2/3}$

$= \left(\frac{5}{2}\right)^{2}$ M1

$= \frac{25}{4}$ A1

2. $\frac{\sqrt{x} \times \sqrt[3]{x}}{x^2}$

$= \frac{x^{1/2} \times x^{1/3}}{x^2}$ M1

$= \frac{x^{5/6}}{x^2}$ A1

$= x^{-7/6}$ A1

3. $9^{x-2} = 27$

$(3^2)^{x-2} = 3^3$ M1

$3^{2x-4} = 3^3$

$2x - 4 = 3$ M1

$2x = 7$

$x = 7/2$ A1

4. $16^x = 4^{1-x}$

$(4^2)^x = 4^{1-x}$ M1

$4^{2x} = 4^{1-x}$

$2x = 1 - x$ M1

$3x = 1 \Rightarrow x = 1/3$ A1

$\boxed{11}$

B2 Surds

1. $\sqrt{72}$

$= \sqrt{36 \times 2}$

$= 6\sqrt{2}$ A1

2. $(2\sqrt{7} - 5\sqrt{3})(3\sqrt{7} + 4\sqrt{3})$

$42 + 8\sqrt{21} - 15\sqrt{21} - 60$ M1 A1

$-7\sqrt{21} - 18$ A1

3. $\frac{11}{2\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$ M1

$= \frac{11\sqrt{5}}{10}$ A1

4. $\frac{8 - 3\sqrt{5}}{2 + \sqrt{5}} \quad \frac{(2 - \sqrt{5})}{(2 - \sqrt{5})}$ M1

$= \frac{16 - 8\sqrt{5} - 6\sqrt{5} + 15}{4 - 5}$ A1 A1

$= \frac{31 - 14\sqrt{5}}{-1} = 14\sqrt{5} - 31$ A1

$\boxed{10}$

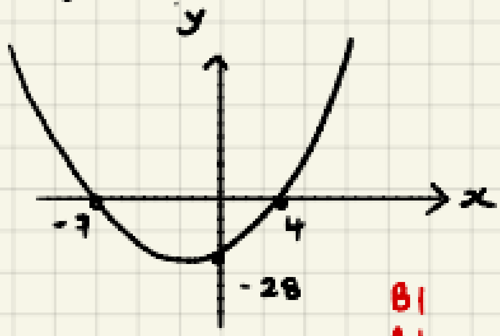
B3 Quadratics

1. (a) (i) $x^2 + 3x - 28 = 0$

$(x+7)(x-4) = 0$ M1

$x = -7$ or $x = 4$ A1

(ii) $y = x^2 + 3x - 28$



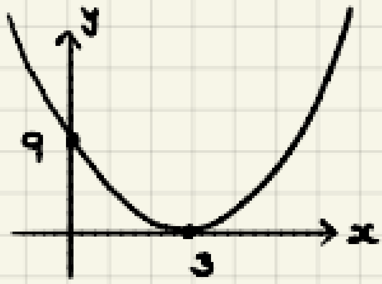
B1
A1
A1

(b) (i) $x^2 - 6x + 9 = 0$

$(x-3)^2 = 0$ M1

A1 $x = 3$ (*repeated*)

(ii) $y = x^2 - 6x + 9$



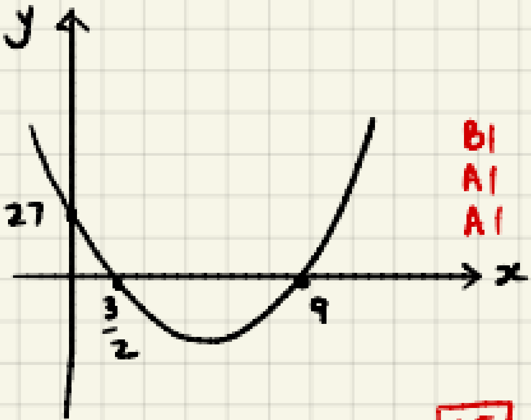
B1
A1
A1

(c) (i) $2x^2 - 21x + 27 = 0$

$(2x-3)(x-9) = 0$ M1

$x = 3/2$ $x = 9$ A1

(ii) $y = 2x^2 - 21x + 27$



B1
A1
A1

$\boxed{15}$

B1 shape, location related to axes

A1 intersections x-axis

A1 intersections y-axis

2. (a) (i) $x^2 + 4x - 7 = 0$

$(x+2)^2 - 4 - 7 = 0$ M1

$(x+2)^2 = 11$

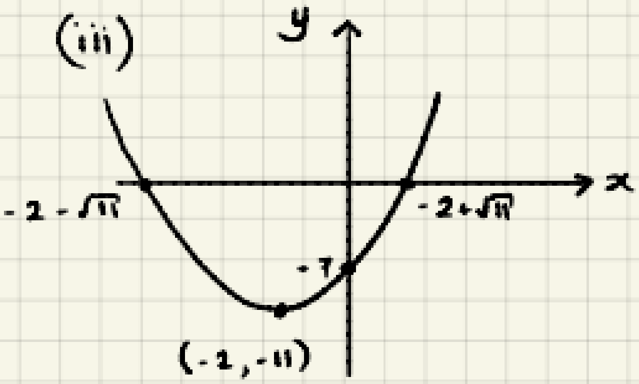
$x + 2 = \pm\sqrt{11}$

$x = -2 \pm \sqrt{11}$ A1

(ii) $y = x^2 + 4x - 7$

$y = (x+2)^2 - 11$ B1

(iii)



Graphs

B1 Shape

A1 Vertex

A1 intersections x-axis

A1 intersections y-axis

(b) (i) $11 + 8x - x^2 = 0$

$-(x^2 - 8x - 11) = 0$ M1

$-[(x-4)^2 - 16 - 11] = 0$ M1

$-(x-4)^2 + 27 = 0$

$(x-4)^2 = 27$

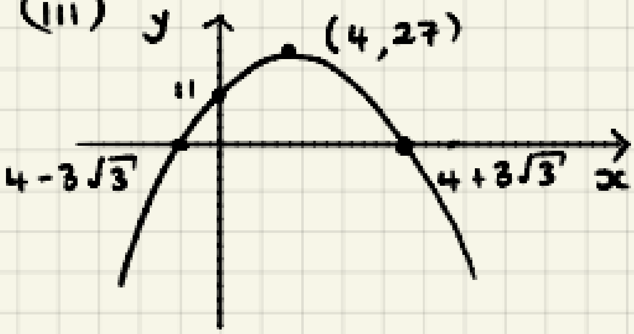
$x - 4 = \pm 3\sqrt{3}$

$x = 4 \pm 3\sqrt{3}$ A1

(ii) $y = 11 + 8x - x^2$

$y = 27 - (x-4)^2$ B1

(iii)



(c) (i) $3x^2 - 12x + 2 = 0$

$3\left[x^2 - 4x + \frac{2}{3}\right] = 0$ M1

$3\left[(x-2)^2 - 4 + \frac{2}{3}\right] = 0$ M1

$3\left[(x-2)^2 - \frac{10}{3}\right] = 0$

$3(x-2)^2 - 10 = 0$

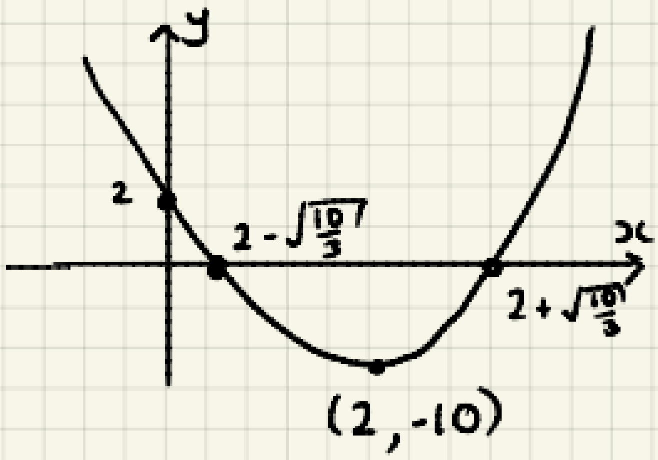
$(x-2)^2 = \frac{10}{3}$

$x - 2 = \pm\sqrt{\frac{10}{3}}$

$x = 2 \pm \sqrt{\frac{10}{3}}$ A1

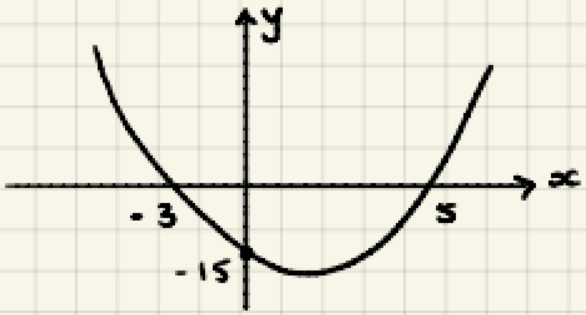
(ii) $y = 3x^2 - 12x + 2$

$y = 3(x-2)^2 - 10$ B1



$\boxed{23}$

3. (a)



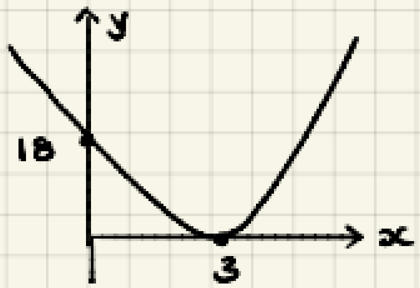
$y = k(x+3)(x-5)$ M1

$-15 = k(3)(-5) \Rightarrow k = 1$ A1

$y = (x+3)(x-5)$

$y = x^2 - 2x - 15$ A1

(b)



$y = k(x-3)^2$ M1

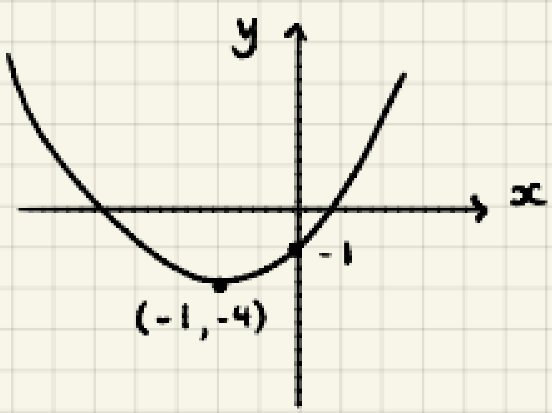
$18 = k(-3)^2 \Rightarrow k = 2$ A1

$y = 2(x-3)^2$

$y = 2(x^2 - 6x + 9)$

$y = 2x^2 - 12x + 18$ A1

(c)



$y = k(x+1)^2 - 4$ M1

(0,-1)

$-1 = k(1)^2 - 4$

$\Rightarrow k = 3$ A1

$y = 3(x+1)^2 - 4$

$y = 3(x^2 + 2x + 1) - 4$

$y = 3x^2 + 6x - 1$ A1

$\boxed{9}$

B4 Simultaneous Equations

1. $3x + 3y = -4$ $\quad$ $6x + 6y = -8$

$5x - 2y = 5$ $\quad$ $15x - 6y = 15$ add $\quad$ M1

$21x = 7$

$x = 1/3$ $\quad$ A1 $\quad$ $3(1/3) + 3y = -4$

$3y = -5$

$x = 1/3,\ y = -5/3$ $\quad$ A1

2. $y = x - 6$

$\frac{1}{2}x - y = 4$

$\frac{1}{2}x - (x - 6) = 4$ $\quad$ M1

$\frac{1}{2}x - x + 6 = 4$

$-\frac{1}{2}x = -2$

$x = 4$ $\quad$ A1 $\quad$ $y = 4 - 6$

$y = -2$

$x = 4,\ y = -2$ $\quad$ A1

3. $3x^2 - x - y^2 = 0$ $\quad x + y = 1$

$3x^2 - x - (1-x)^2 = 0$ M1 $\quad y = 1 - x$

$3x^2 - x - (1 - 2x + x^2) = 0$

$3x^2 - x - 1 + 2x - x^2 = 0$

$2x^2 + x - 1 = 0$ A1

$(2x-1)(x+1) = 0$

$x = 1/2$ $\quad x = -1$ A1

$y = 1 - 1/2$ $\quad y = 1 - -1$

$x = 1/2 \;\; y = 1/2$ A1 $\quad x = -1, y = 2$ A1

$\boxed{11}$

B5 Inequalities

1. $3(1-2t) \leq t - 4$

$3 - 6t \leq t - 4$ M1

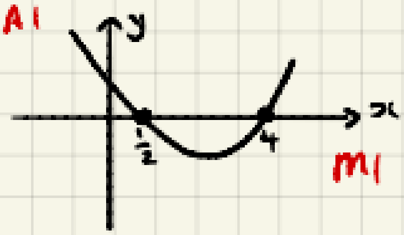
$7 \leq 7t$

$t \geq 1$ A1

2. $2x^2 - 9x + 4 \leq 0$

$(2x-1)(x-4) \leq 0$ M1

CVs $x = 1/2 \quad x = 4$ A1



M1

$\frac{1}{2} \leq x \leq 4$ A1

3. $2y + 3 < 3y(y-2)$

$2y + 3 < 3y^2 - 6y$

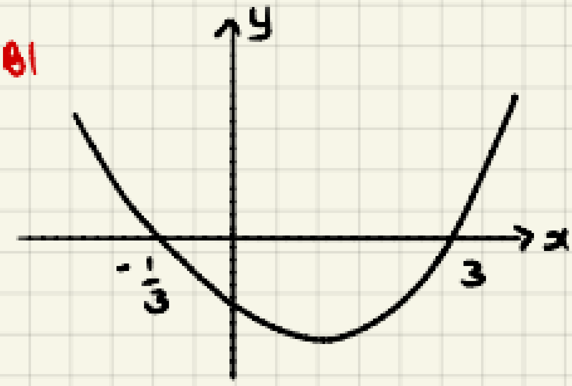
$0 < 3y^2 - 8y - 3$ B1

$\boxed{11}$

$3y^2 - 8y - 3 > 0$

$(3y+1)(y-3) > 0$ M1

CVs $y = -1/3 \quad y = 3$ A1

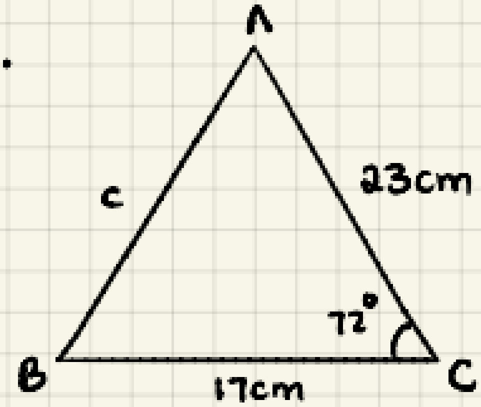


M1

$y < -1/3$ or $y > 3$ A1

E1 Triangle Geometry

1.



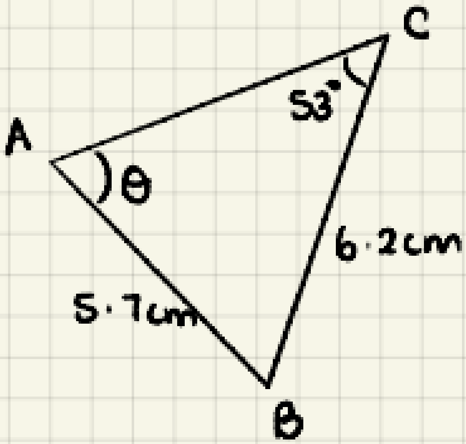
$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 17^2 + 23^2 - 2(17)(23)\cos 72°$ M1

$c^2 = 576.35$

$AB = 24.0\text{cm}$ A1

2.

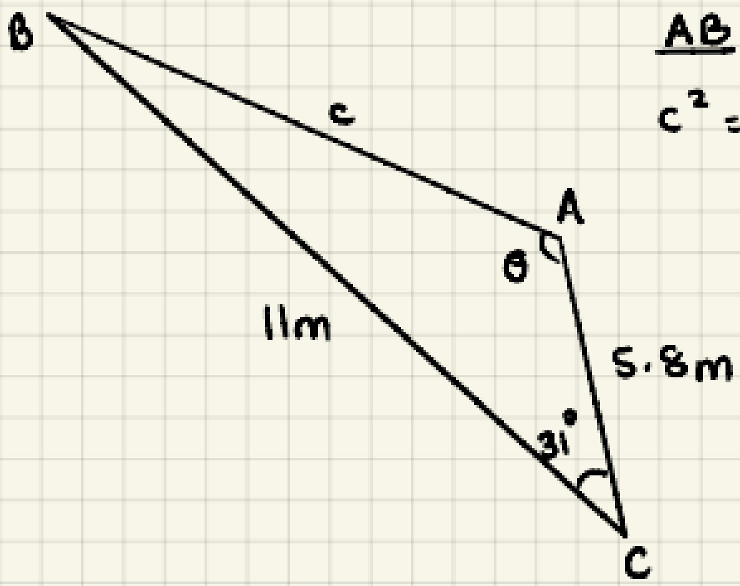


$\frac{\sin\theta}{6.2} = \frac{\sin 53}{5.7}$ M1

$\theta = \sin^{-1}\left(\frac{6.2\sin 53}{5.7}\right)$

$\theta = 60.3°$ A1

3.



AB

$c^2 = 5.8^2 + 11^2 - 2(5.8)(11)\cos 31$

$c^2 = 45.27$ M1

$AB = 6.7\text{m}$ A1

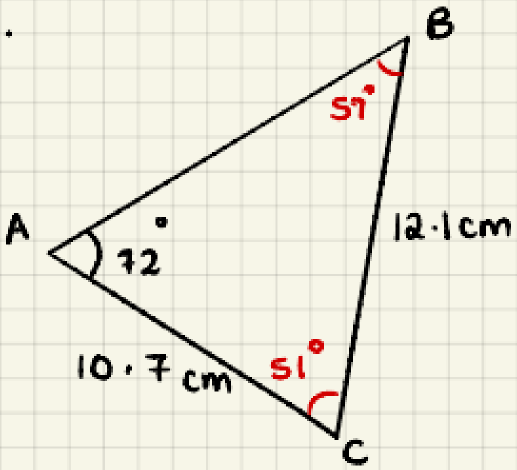
θ

M1 $\cos\theta = \frac{5.8^2 + 6.7^2 - 11^2}{2(5.8)(6.7)}$

$\theta = \cos^{-1}(-0.546)$

A1 $\theta = 123°$

4.



$$\frac{\sin B}{10.7} = \frac{\sin 72°}{12.1}$$

M1

$$B = \sin^{-1}\left(\frac{10.7 \sin 72°}{12.1}\right)$$

$\theta = 57°$ A1

$A = \frac{1}{2} ab \sin C$

M1

$= \frac{1}{2}(10.7)(12.1) \sin 51°$

$= 50.3 cm^2$ A1

$\boxed{12}$

Re-arranging equations

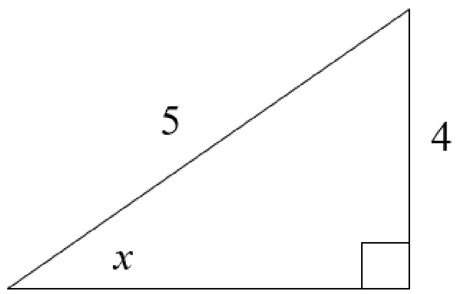
1.	To find velocity, v, we use the formula $v^2 = u^2 + 2as$ Rearrange to find s →	$v^2 - u^2 = 2as$ $s = \dfrac{v^2 - u^2}{2a}$
2.	Make x the subject of $4F = F + \dfrac{a}{y+x}$ →	$3F = \dfrac{a}{y+x}$ $3Fy + 3Fx = a$ $3Fx = a - 3Fy$ $x = \dfrac{a - 3FY}{3F}$

E7 Trigonometric equations

Solve each equation for θ in the interval $0 \le \theta \le 360°$ giving your answers to 1 decimal place.			
1.	$\theta = 66.4,\ 360 - 66.4$ $\theta = 66.4°,\ 293.6°$	**2.**	$2x = 30,\ 180 - 30,$ $360 + 30,\ 540 - 30$ $= 30,\ 150,\ 390,\ 510$ $x = 15,\ 75,\ 195,\ 255$

E3 Exact Trigonometric values

1.



Now Pythagoras's Theorem tells you that the third, adjacent, side is 3 $\sqrt{5^2 - 4^2} = 3$

Hence using SOH, CAH, TOA, $\cos x° = \frac{3}{5}$

$\tan x° = \dfrac{4}{3}$

Year 12 Initial Test for Mathematics

Write out the solutions to each of the following questions.
Show full working, ***without*** *the use of a calculator.*

Practice 2 (No Calculator)

B1 Indices

1.	Evaluate $\left(3\frac{3}{8}\right)^{-1/3}$	**2.**	Express in the form x^k $\frac{\sqrt{x} \times \sqrt[5]{x}}{x^2}$	**3.**	Solve $3^{3x-2} = \sqrt[3]{9}$	**4.**	Solve $\left(\frac{1}{2}\right)^{1-x} = \left(\frac{1}{8}\right)^{2x}$

B2 Surds

1.	Simplify $\sqrt{80}$	**2.**	Expand and simplify $(7 - 3\sqrt{5})\,(3\sqrt{5} - 2)$	**3.**	Rationalise the denominator $\frac{7}{5\sqrt{3}}$	**4.**	Rationalise the denominator $\frac{3 + 5\sqrt{11}}{7 - \sqrt{11}}$

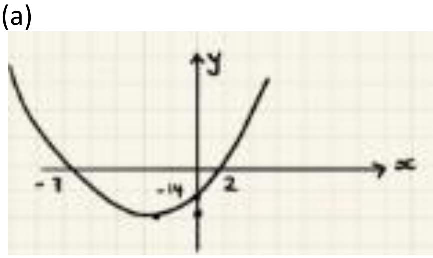
B3 Quadratics

1. Solve the following quadratic equations by factorising and use your solutions to sketch the related quadratic graph, labelling all intersections with the coordinate axis.		
(a) (i) $x^2 - 13x + 40 = 0$	(b) (i) $x^2 + 5x = 0$	(c) (i) $6x^2 + 5x - 4 = 0$
(a) (ii) Sketch $y = x^2 - 13x + 40$	(b) (ii) Sketch $y = x^2 + 5x$	(c) (ii) Sketch $y = 6x^2 + 5x - 4$

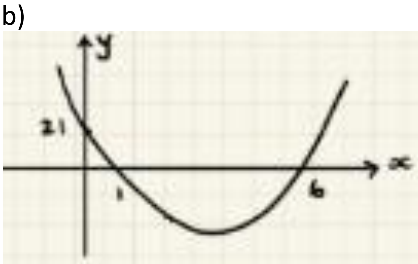
2. Solve the following quadratic equations by completing the square and use your solutions to sketch the related quadratic graph, labelling all intersections with the coordinate axis and turning point.		
(a) (i) $x^2 + 2x - 20 = 0$	(b) (i) $-11 + 8x - x^2 = 0$	(c) (i) $3x^2 - 18x + 2 = 0$
(ii) Write $y = x^2 + 2x - 20$ in the form $y = a(x + b)^2 + c$	(ii) Write $y = -11 + 8x - x^2$in the form $y = a(x + b)^2 + c$	(ii) Write $y = 3x^2 - 18x + 2$ in the form $y = a(x + b)^2 + c$
(iii) Sketch $y = x^2 + 2x - 20$	(iii) Sketch $y = -11 + 8x - x^2$	(iii) Sketch $y = 3x^2 - 18x + 2$

3. Evaluate the equation of the following quadratics, giving your answer in the form $y = ax^2 + bx + c$

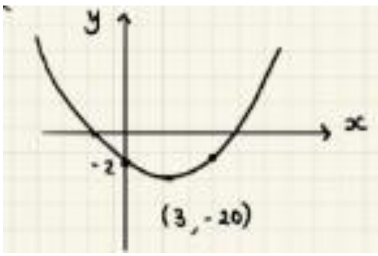
(a)



(b)



(c)



B4 Simultaneous Equations

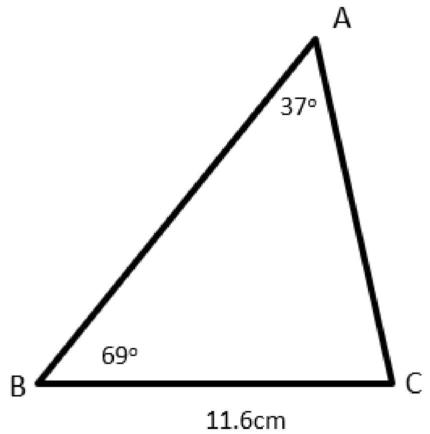
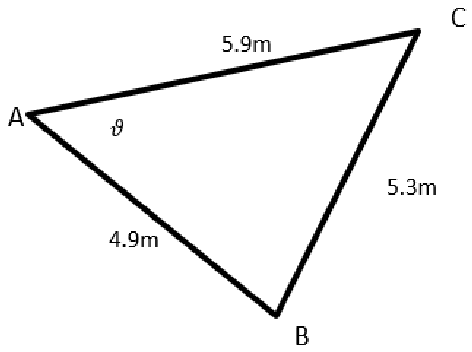
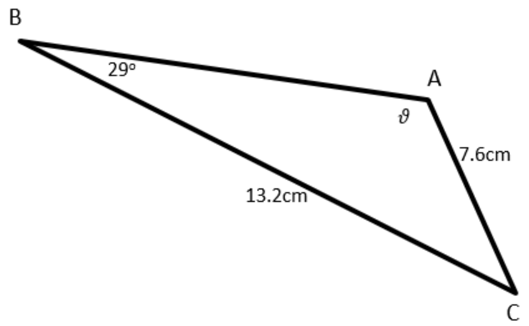
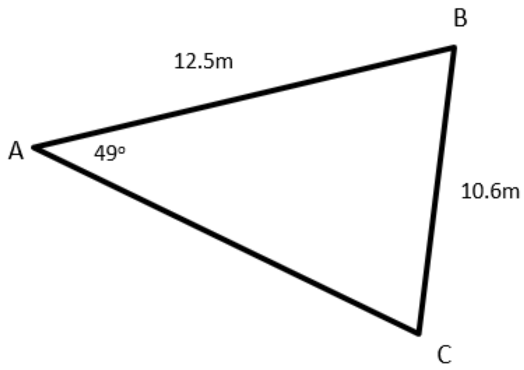
1.	Solve $$3x - 4y = 16$$ $$2x + 12y = 7$$	**2.**	Solve $$3y = 2x - 8$$ $$4x + y = -5$$	**3.**	Solve $$3x^2 - xy + y^2 = 36$$ $$x - 2y = 10$$

B5 Inequalities

Find the set of values for which…

1.	$4(5 - 2y) \geq 3(7 - 2y)$	**2.**	$2x^2 - 5x - 3 > 0$	**3.**	$x(2x + 1) \leq x^2 + 6$

E1 Triangle Geometry (Calculator)

1.	Calculate the length AB 	**2.**	Calculate the angle ϑ 
3.	Calculate the length AB and the obtuse angle ϑ 	**4.**	Calculate the area of the triangle ABC 

Re-arranging equations

1.	Make x the subject of $x + a = \frac{x+b}{c}$	**2.**	Make a the subject of $\frac{1-a}{1+a} = \frac{x}{y}$

E7 Trigonometric equations

Solve each equation for θ in the interval $0 \le \theta \le 360°$ giving your answers to 1 decimal place.			
1.	$\tan\theta = 1.6$	**2.**	$\cos 2x° = 0.64$

E3 Exact Trigonometric values

Find the exact values of $\cos x$ and $\tan x$ given that:

$\sin x = \dfrac{1}{3}$ and $0^\circ < x < 90^\circ$

Practice Test 2

B1 Indices

1. $\left(3\frac{3}{8}\right)^{-1/3} = \left(\frac{27}{8}\right)^{-\frac{1}{3}}$ M1

$= \left(\frac{8}{27}\right)^{1/3}$

$= \frac{2}{3}$ A1

2. $\frac{\sqrt{x} \times \sqrt[5]{x}}{x^2}$

$= \frac{x^{1/2} \times x^{1/5}}{x^2}$ M1

$= \frac{x^{7/10}}{x^2}$ A1 $= x^{-13/10}$ A1

3. $3^{3x-2} = \sqrt[3]{9}$

$3^{3x-2} = (3^2)^{1/3}$ M1

$3^{3x-2} = 3^{2/3}$

$3x - 2 = \frac{2}{3}$ M1

$3x = \frac{8}{3} \Rightarrow x = \frac{8}{9}$ A1

4. $\left(\frac{1}{2}\right)^{1-x} = \left(\frac{1}{8}\right)^{2x}$

$(2^{-1})^{1-x} = (2^{-3})^{2x}$ M1

$2^{-1+x} = 2^{-6x}$

$-1 + x = -6x$ M1

$7x = 1$

$x = 1/7$ A1

$\boxed{11}$

B2 Surds

1. $\sqrt{80}$

$= \sqrt{16 \times 5}$

$= 4\sqrt{5}$ A1

2. $(7 - 3\sqrt{5})(3\sqrt{5} - 2)$

$= 21\sqrt{5} - 14 - 45 + 6\sqrt{5}$ M1 A1

$= 27\sqrt{5} - 59$ A1

3. $\frac{7}{5\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ M1

$= \frac{7\sqrt{3}}{15}$ A1

4. $\frac{3 + 5\sqrt{11}}{7 - \sqrt{11}} \; \frac{(7 + \sqrt{11})}{(7 + \sqrt{11})}$ M1

$= \frac{21 + 3\sqrt{11} + 35\sqrt{11} + 55}{49 - 11}$ A1 A1

$= \frac{76 + 38\sqrt{11}}{38}$

$= 2 + \sqrt{11}$ A1

$\boxed{10}$

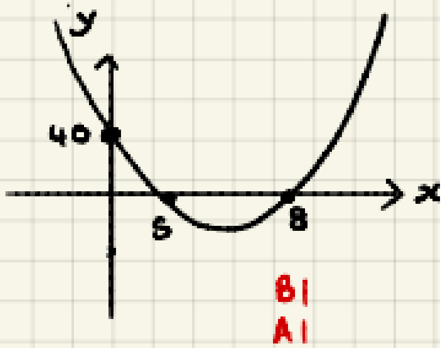
B3 Quadratics

1. (a) (i) $x^2 - 13x + 40 = 0$

$(x-8)(x-5) = 0$ M1

$x = 8 \quad x = 5$ A1

(ii) $y = x^2 - 13x + 40$



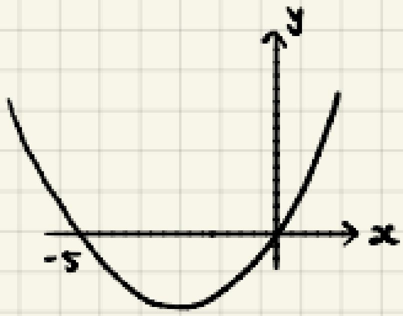
B1
A1
A1

(b) (i) $x^2 + 5x = 0$

$x(x+5) = 0$ M1

$x = 0 \quad x = -5$ A1

(ii) $y = x^2 + 5x$



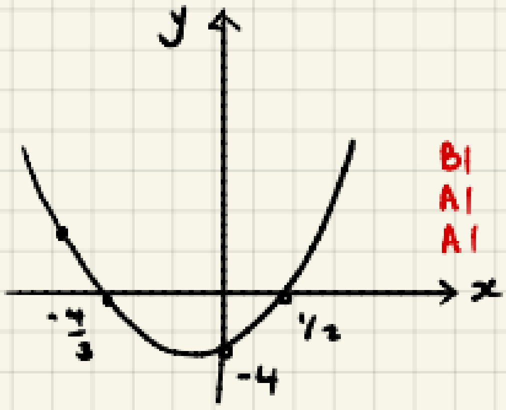
B1
A1
A1

(c) (i) $6x^2 + 5x - 4 = 0$

$(3x+4)(2x-1) = 0$ M1

$x = -4/3 \quad x = 1/2$ A1

(ii) $y =$



B1
A1
A1

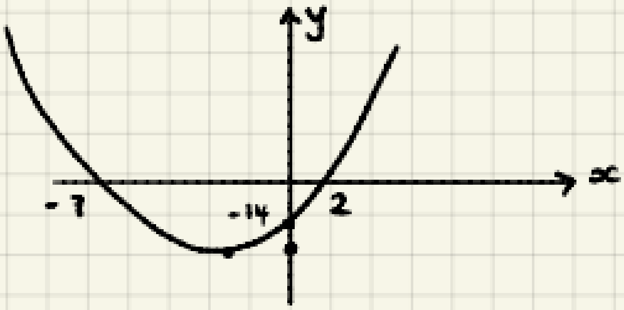
$\boxed{15}$

B1 shape, location related to axes

A1 intersections x-axis

A1 intersections y-axis

3. (a)



$y = k(x+7)(x-2)$ M1

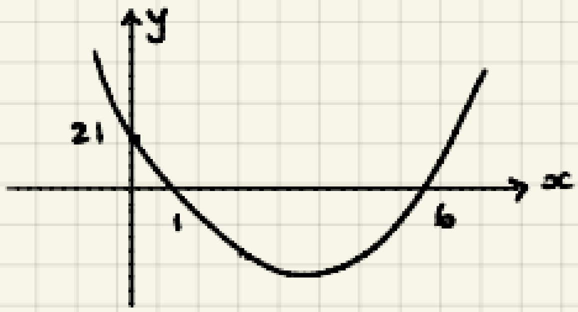
$-14 = k(7)(-2)$

$k = 1$ A1

$y = (x+7)(x-2)$

$y = x^2 + 5x - 14$ A1

(b)



$y = k(x-1)(x-6)$ M1

$21 = k(-1)(-6)$

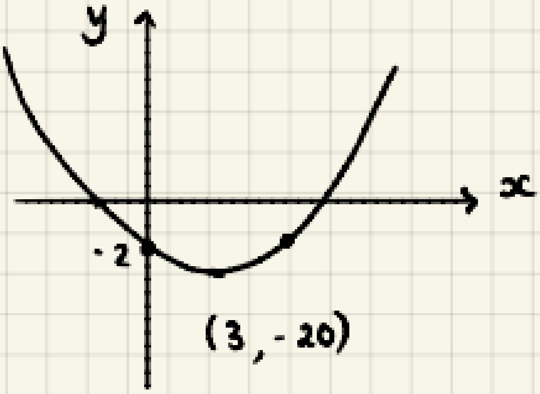
$\Rightarrow k = \frac{21}{6} = \frac{7}{2}$ A1

$y = \frac{7}{2}(x-1)(x-6)$

$y = \frac{7}{2}(x^2 - 7x + 6)$

$y = \frac{7x^2}{2} - \frac{49x}{2} + 21$ A1

(c)



$y = k(x-3)^2 - 20$ M1

$-2 = k(-3)^2 - 20$

$18 = k(9)$

$k = 2$ A1

$y = 2(x-3)^2 - 20$

$y = 2(x^2 - 6x + 9) - 20$

$y = 2x^2 - 12x - 2$ A1

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B4. Simultaneous equations

1. $3x - 4y = 16$ $\qquad$ $9x - 12y = 48$

$2x + 12y = 7$ $\qquad$ $\underline{2x + 12y = 7}$ $\qquad$ M1

$11x = 55$

$x = 5$ $\quad$ A1

$3x - 4y = 16$
$15 - 4y = 16$
$-1 = 4y$
$y = -1/4$

$x = 5, y = -1/4$ $\quad$ A1

2. $3y = 2x - 8$ $\Rightarrow$ $2x = 3y + 8$
$4x = 6y + 16$

$4x + y = -5$ $\qquad$ M1

$6y + 16 + y = -5$

$7y = -21$

$y = -3$ $\qquad$ $2x = 3y + 8$
$2x = 3(-3) + 8$
$x = -1/2$ $\quad$ A1 $\qquad$ $x = -1/2$ $y = -3$ A1

3. $3x^2 - xy + y^2 = 36$

$x - 2y = 10$ $\Rightarrow$ $x = 2y + 10$

$3(2y+10)^2 - (2y+10)y + y^2 = 36$ $\qquad$ M1

$3(4y^2 + 40y + 100) - y(2y+10) + y^2 = 36$

$12y^2 + 120y + 300 - 2y^2 - 10y + y^2 = 36$

$11y^2 + 110y + 264 = 0$

$y^2 + 10y + 24 = 0$ $\qquad$ A1

$(y+6)(y+4) = 0$

M1

$y = -6$ $\qquad$ $y = -4$

$x = 2(-6) + 10$ $\qquad$ $x = 2(-4) + 10$

$\boxed{11}$

$x = -2$ $\qquad$ $x = 2$

$x = -2, y = -6$ A1 $\qquad$ $x = 2, y = -4$ A1

B5 Inequalities

1. $4(5-2y) \geqslant 3(7-2y)$

$20 - 8y \geqslant 21 - 6y$ M1

$-1 \geqslant 2y$

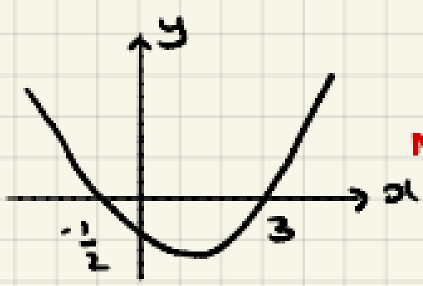
$-\frac{1}{2} \geqslant y$

$y \leq -\frac{1}{2}$ A1

2. $2x^2 - 5x - 3 > 0$

$(2x+1)(x-3) > 0$ M1

CVs $x = -\frac{1}{2}$ $x = 3$ A1



M1

$x < -\frac{1}{2}$ or $x > 3$ A1

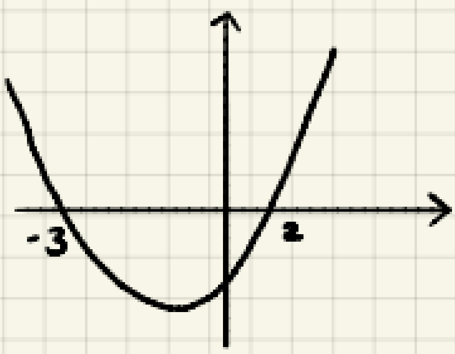
3. $x(2x+1) \leq x^2 + 6$

$2x^2 + x \leq x^2 + 6$ M1

$x^2 + x - 6 \leq 0$

$(x+3)(x-2) \leq 0$ M1

CVs $x = -3$ $x = 2$ A1



M1

$-3 \leq x \leq 2$ A1

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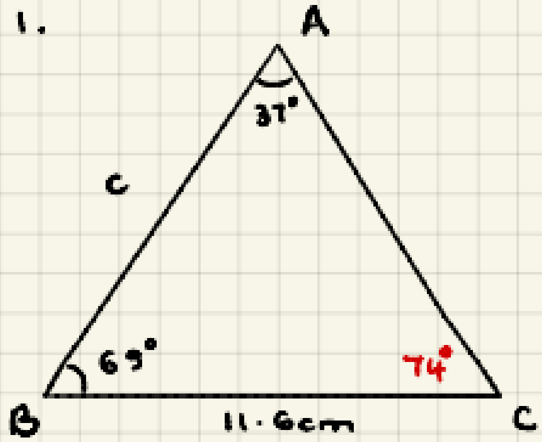
E1 Triangle Geometry

1.

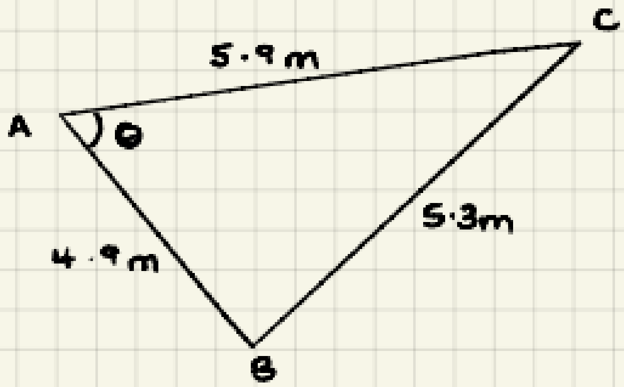


$$\frac{c}{\sin 74°} = \frac{11.6}{\sin 37°}$$

$$c = \frac{11.6 \sin 74°}{\sin 37°}$$ M1

$c = 18.5\text{cm}$ A1

2.



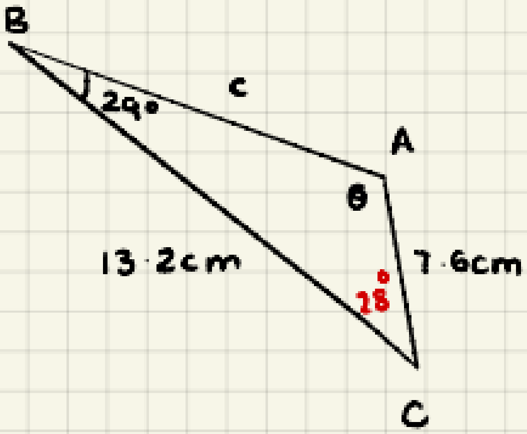
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos\theta = \frac{5.9^2 + 4.9^2 - 5.3^2}{2(5.9)(4.9)}$$ M1

$\cos\theta = 0.53148$

$\theta = 57.9°$ A1

3.



θ

$$\frac{\sin\theta}{13.2} = \frac{\sin 29}{7.6}$$

$$\sin\theta = \frac{13.2 \sin 29°}{7.6}$$ M1

$\sin\theta = 0.8420$

$\theta = 57.4°$

obtuse $\Rightarrow \theta = 123°$ A1

AB

$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 13.2^2 + 7.6^2 - 2(13.2)(7.6)\cos 28$ M1

$c^2 = 54.8 \Rightarrow c = 7.4\text{cm}$ A1

Re-arranging equations

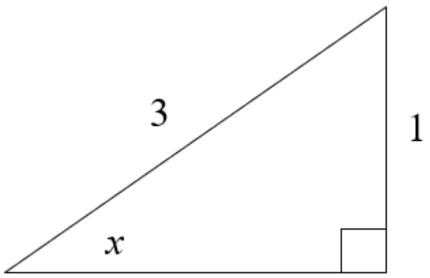
1.	Make x the subject of $x + a = \frac{x+b}{c}$ →	$c(x+a) = x + b$ $cx + ca - x = b$ $cx - x = b - ca$ $x(c-1) = b - ca$ $x = \dfrac{b - ca}{c - 1}$
2.	Make a the subject of $\frac{1-a}{1+a} = \frac{x}{y}$ →	$y(1-a) = x(1+a)$ $y - ay = x + xa$ $y - x = xa + ay$ $a(x+y) = y - x$ $a = \dfrac{y - x}{x + y}$

E7 Trigonometric equations

Solve each equation for θ in the interval $0 \le \theta \le 360°$ giving your answers to 1 decimal place.			
1.	$\theta = 66.4,\ 360 - 66.4$ $\theta = 66.4°,\ 293.6°$	**2.**	$2x = 30,\ 180 - 30,$ $360 + 30,\ 540 - 30$ $= 30,\ 150,\ 390,\ 510$ $x = 15,\ 75,\ 195,\ 255$

E3 Exact Trigonometric values

1.



Now Pythagoras's Theorem tells you that the third, adjacent, side is 3 $\sqrt{3^2 - 1^2} = \sqrt{8}$

Hence using SOH, CAH, TOA, $\cos x° = \frac{\sqrt{8}}{3}$ or $\frac{2\sqrt{2}}{3}$

$\tan x° = \dfrac{\sqrt{2}}{4}$